

# Compact Electro-thermal Model of Semiconductor Device with Nonlinear Convection Coefficient

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## Abstract

Compact thermal models for semiconductor devices are usually constructed under the assumption of constant material properties as well as the convection coefficient. However, in many cases this is the cause of undesired deviations from experimental results. In this paper, we present an extension of model order reduction to construct a compact thermal model for the case when a convection coefficient is nonlinear. As an example, we use transient thermal simulation of a semiconductor device with a temperature-dependent convection coefficient. We demonstrate that the model order reduction technique introduced in this paper can treat such nonlinearity very efficiently. At the same time, the reduced model is accurate enough to replace the original model.

## 1. Introduction

Engineers often employ compact thermal models to speed up system-level simulation. However, conventional compact models are limited to the linear case when all material properties are assumed to be constant [1, 2]. This may not be enough to describe accurately measured results. There was an attempt to generalize conventional compact thermal modeling to include nonlinear behavior [3]. However, because the construction of compact models is based on data fitting, the problem to choose the right topology for a compact model becomes even more difficult.

Formal model reduction [4, 5] takes as input high-dimensional system matrices of ordinary differential equations obtained after the discretization in space of the heat transfer equation. Then, it generates automatically an accurate compact model [6, 7] by means of a formal procedure based on matrix linear algebra. Unfortunately, this is also limited to the linear case.

Recently, there were successful attempts to extend the model reduction approach to a nonlinear system: see review [8] and case studies related to an electrical circuit [9] and MEMS [10]. In the present paper, we use a thermal model for a semiconductor device [6] as a case study to apply these methods to the problem when the film coefficient depends on temperature.

In the following text, we first give a review of nonlinear model order reduction [3] in section 2. The nonlinear model of a semiconductor device and the application of model order reduction are described in section 3. In section 4, we show the efficiency of nonlinear model order reduction with simulation results. Conclusions are given in section 5.

## 2. Overview of nonlinear model order reduction

In this section, we overview the model order reduction method from [9]. We describe its application in the next section.

The nonlinear system considered in [9] is of the following form,

$$\left. \begin{aligned} dx/dt &= f(x) + bu(t) \\ y(t) &= c^T x(t) \end{aligned} \right\} \quad (1)$$

where the vector  $x(t) \in R^n$  is often referred to as a "state variable". The dimension  $n$  of this vector is usually called the dimension of the system. The initial condition is assumed to be  $x_0 = 0$ ,  $u(t)$  is the input signal,  $y(t)$  is the output response, a linear combination of the state vector entries. From an engineering viewpoint, the output is useful information that can be directly obtained from the state vector.

Let us assume that  $f(x)$  is smooth enough so that it can be expanded into a Taylor series about the initial condition, for example

$$f(x) = D_f(0)x + x^T H_f(0)x + \dots \quad (2)$$

where  $D_f(0)$  is the Jacobian matrix of  $f(x)$  at 0,  $H_f(0)$  is the so called Hessian tensor, for detailed explanations of  $H_f(0)$  see [9].

If  $f(x)$  was approximated by the first two components of the Taylor expansion above, then the quadratic nonlinear system below is an approximation to the original nonlinear system (1), provided  $A = D_f(0)$ ,  $W = H_f(0)$ ,

$$\left. \begin{aligned} dx/dt &= Ax + x^T Wx + bu(t) \\ y &= c^T x(t) \end{aligned} \right\} \quad (3)$$

The projection matrix for model reduction is computed by

$$\text{colspan}\{\tilde{V}\} = \text{colspan}\{A^{-1}b, A^{-2}b, \dots, A^{-q}b\} \quad (4)$$

This equation is inspired by linear order reduction based on the Krylov subspace projection technique [4]. The terms on the right hand side of (4) are the moment vectors of the transfer function of the linear system below, which only contains the linear part in (3)

$$\left. \begin{aligned} dx/dt &= Ax + bu(t) \\ y &= c^T x(t) \end{aligned} \right\} \quad (5)$$

The columns in  $\tilde{V}$  are required to be orthogonal with each other, that is  $\tilde{V}^T \tilde{V} = I$ ,  $I$  being the identity matrix. In (4), on the left hand side, *colspan* means the subspace spanned by  $\tilde{V}$ , on the right hand side, *colspan* means the subspace spanned by  $A^{-1}b, A^{-2}b, \dots, A^{-q}b$ .

When  $\tilde{V}$  is applied to the quadratic system (3) by means of the approximation  $x \approx \tilde{V}z$ , Eq (3) becomes

$$\left. \begin{aligned} \tilde{V}dz/dt &= A\tilde{V}z + g(z) + bu(t) \\ y &= c^T \tilde{V}z(t) \end{aligned} \right\} \quad (6)$$

where  $g(z) = (\tilde{V}z)^T W \tilde{V}z$ . After the multiplication of the first equation in (6) by  $\tilde{V}^T$  on both sides, we obtain

$$\left. \begin{aligned} \tilde{V}^T \tilde{V}dz/dt &= \tilde{V}^T A\tilde{V}z + \tilde{V}^T g(z) + \tilde{V}^T bu(t) \\ y &= c^T \tilde{V}z(t) \end{aligned} \right\} \quad (7)$$

With a new notation  $\hat{A} = \tilde{V}^T A\tilde{V}$ ,  $\hat{g} = \tilde{V}^T g$ ,  $\hat{b} = \tilde{V}^T b$ , (7) can be simplified into (8) which is the final reduced model.

$$\left. \begin{aligned} dz/dt &= \hat{A}z + \hat{g}(z) + \hat{b}u(t) \\ y &= c^T \tilde{V}z(t) \end{aligned} \right\} \quad (8)$$

In order to obtain the original solution  $x(t)$  in (1) from this reduced model (8), we merely solve (8) and obtain the solution  $z(t)$ , then use  $\tilde{x} = \tilde{V}z$  as the approximate solution for  $x(t)$ . If the approximation  $\tilde{x} = \tilde{V}z$  is accurate enough, the original large model in (1) can be replaced by the small model (8) during simulation, and this can save a lot of simulation time and memory without loss of accuracy.

A circuit example is given in [9] to show the efficiency of this model order reduction technique. However, this circuit example is only of moderate dimension  $n=100$ . In the next section, we will introduce a nonlinear model with the dimension of  $n=67112$  and show the applicability of the nonlinear model order reduction technique for a high dimensional model.

### 3. Case study: a large-scale thermal model of a semiconductor device

We have made the finite element model of a semiconductor device similar to that presented in [6] (see Fig. 1). The temperature-dependent convection coefficient

$$h(t) = a + bT$$

was applied with the convection boundary conditions at the surface. After the discretization, we had a system of ordinary differential equations as follows,

$$\left. \begin{aligned} Edx/dt + Ax + f(x) &= B \\ y(t) &= Cx(t) \end{aligned} \right\} \quad (9)$$

where  $y(t)$  is the temperature at the specific location in the device.  $f(x)$  is a nonlinear function that contains the temperature-dependent convection coefficient. It is already a quadratic function of  $x$ , hence, we did not need to make its Taylor expansion. The model in (9) is of the same form as in (3), i.e., it only contains the linear term and the quadratic term.

The dimension of (9) was 67112. The system matrices in (9) were obtained from ANSYS binary EMAT files by means of mor4ansys [11].

The projection matrix  $V$  was constructed from (9) directly. According to (4) and (5), we first derived the corresponding linear system of (9),

$$\left. \begin{aligned} Edx/dt + Ax &= B \\ y(t) &= Cx(t) \end{aligned} \right\} \quad (10)$$

The transfer function of (10) is as follows

$$H(s) = C(sE + A)B = \sum_{i=0}^{\infty} C(A^{-1}E)^i A^{-1}Bs^i \quad (11)$$

The moment vector of  $H(s)$  are the terms in the series expansion above, and the projection matrix  $V$  is thus constructed as follows,

$$\left. \begin{aligned} colspan\{V\} &= \\ colspan\{A^{-1}b, (A^{-1}E)A^{-1}b, \dots, (A^{-1}E)^q A^{-1}b\} \end{aligned} \right\} \quad (12)$$

By projecting the original system (8) onto the subspace spanned by  $V$  (similar to 6 and 7), we obtained the reduced system of (8) in the next form,

$$\left. \begin{aligned} \hat{E}dz/dt + \hat{A}z + \hat{f}(Vz) &= \hat{B} \\ \hat{y}(t) &= \hat{C}z(t) \end{aligned} \right\} \quad (13)$$

where  $\hat{E} = V^T E V$ ,  $\hat{A} = V^T A V$ ,  $\hat{f}(z) = V^T f(Vz)$ ,  $\hat{B} = V^T B$ ,  $\hat{C} = C V$ . The dimension of the system (13) was 300.

In the next section, we will show the efficiency of the reduced model (13) with simulation results, when simulation of the original large model (8) with the dimension of 67112 is compared with simulation of (13) with the dimension of 300.

#### 4. Simulation results

In this section, we compare the simulation results between the reduced model (13) and the original large model (8) in figure 2 and figure 3 that shows the temperatures at two locations within the device. In each figure, the solid line is the solution  $y(t)=Cx(t)$  computed directly from the original large model. Its simulation was done in ANSYS. The dashed line is the approximate solution computed from the reduced model (13) by means of  $y(t) \approx \hat{y}(t) = \hat{C}\hat{x} = Vz$ . Its simulation was done in MATLAB. The agreement, and thus the accuracy of the reduced model, is very good for different positions within the device.

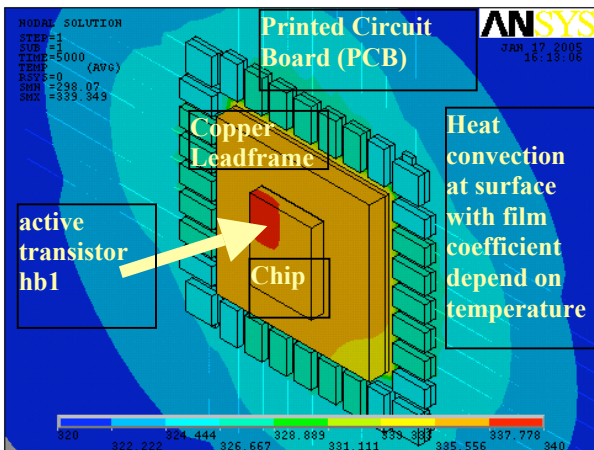


Figure 1: Semiconductor device with temperature distribution

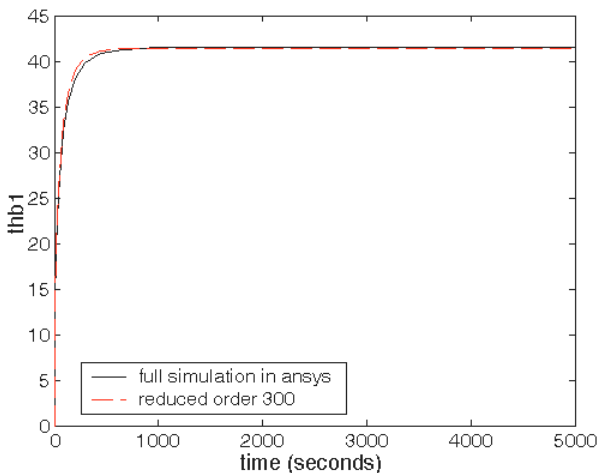


Figure 2: Comparison of the temperature rise at the location thb1.

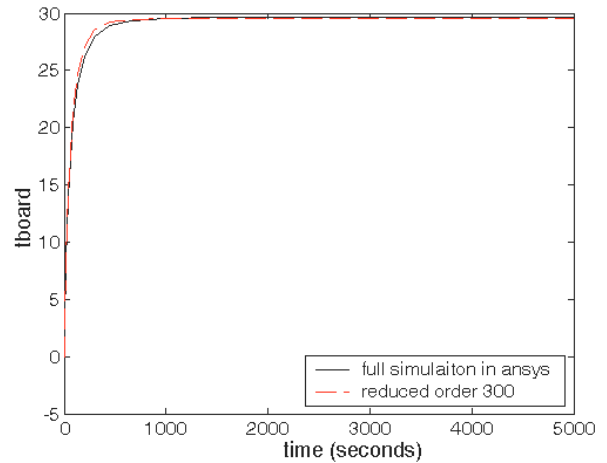


Figure 3: Comparison of the temperature rise at the location tboard.

#### 5. Conclusions

We have introduced a nonlinear model order reduction technique to produce fast simulation of a very large-scale system derived from a thermal model of a semiconductor device. The simulation results show that the reduced small model is accurate enough to replace the original large model so that only the small model needs to be simulated. This saves simulation time and computer memory. The results confirm the applicability of model order reduction technique to a high-dimensional weakly nonlinear system.

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