Parameter preserving Model Order Reduction of a Flow Meter

C. Moosmann*, E. B. Rudnyi*, A. Greiner*, J. G. Korvink*, M. Hornung**

* IMTEK, University of Freiburg, Germany, moosmann@imtek.de, korvink@imtek.de
** Sensirion AG, Zurich, Switzerland

ABSTRACT

Model order reduction techniques are known to work reliably for finite-element-type simulations of MEMS devices. These techniques can tremendously shorten computational times for transient and harmonic analyses. However, standard model reduction techniques cannot be applied if the equation system incorporates time-varying matrices or parameters that are to be preserved for the reduced model. At the same time, if one aims at automatic compact model generation it is most likely that there are parameters that have to be preserved.

In this paper we demonstrate a method, based on a multivariant Padé-type expansion, that can preserve scalar parameters or functions during the reduction process. We show that this method is applicable to convection-diffusion type problems. As a technical example we investigate a micro anemometer.

Keywords: parameter, model order reduction, flow meter, heat transport, convection

1 Considered application

The application to demonstrate the proposed reduction technique is an anemometer. It should be stated that the method itself is not limited to a specific equation type, but that it is also possible to reduce problems from other physical domains using this technique.

An anemometer is a flow meter that consists of a heater and temperature sensors before and after the heater in the direction of the flow. The flow influences the temperature field and thus leads to a temperature difference between the sensors. This temperature difference is measured and used to determine the fluid flow [1].

1.1 Model generation

An important operating condition for anemometers is that the heat inserted into the fluid is so small that it does not change the flow significantly and, of course that it does not cause chemical reactions in the fluid. For modeling the anemometer we can especially use the first condition very effectively. With this condition we can exclude the computation of the fluid flow from the model, and take it as specified. This has the great advantage, in that the equation to solve is only a linear convection-diffusion problem, instead of the full set of non-isothermal Navier-Stokes equations. This approach is often referred to as the forced-convection approach, because natural convection is not taken into account.

To generate and discretize the model we use the ANSYS simulation environment [2]. This multi-physics finite element environment offers the forced convection approach. Figure 1 shows the geometry of the model. At the bottom, there is the chip with the heater and the sensors residing on a membrane, which functions also as the wall of the channel. The fluids above and below the membrane are assumed to have the same properties, however, no convection is applied below. Since the temperature field usually does not dissipate far into the flow field, only half of the channel is modeled. Another important aspect is the cascaded flow profile. While a parabolic profile would be best suited, ANSYS can only handle constant velocities during the discretization. Therefore the channel area was divided into four areas with constant velocity, leading to an approximation of the real flow profile. We want to stress again that this approximation is not imposed by the model reduction process, but by the discretization possibilities of ANSYS.

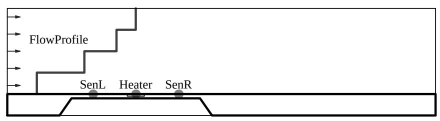

Figure 1: Model of the anemometer

The forced convection approach requires a rather fine meshing, in this case 61655 elements and 31200 nodes; after applying the Dirichlet boundary conditions leading to 29008 degrees of freedom for the 2D problem. If a real 3D structure is to be simulated the dimensionality of the original model would be remarkably higher. That is, because the maximum mesh size is dependent on the convection term. So if the convection term increases an equally more refined mesh is needed. This dependency can be expressed in the grid Pecklet number as described in [3].

As the focus of this paper is the order reduction process we will not discuss the validity of the model in more detail. The original discretization will be used as a reference. However, we shortly want to summarize the assumptions made during the modeling:

- The heat flow does not influence the fluid flow, therefore no natural convection can be modeled.
- Material parameters are assumed to be constant.
- A cascaded flow-profile is used.

1.2 Mathematical formulation of the considered problem

The physical model is a standard convection-diffusion problem (eqn 1).

$$\rho c \cdot \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) - \rho c \cdot \vec{\mathbf{v}} \nabla T + \dot{q} \tag{1}$$

,where ρ denotes the mass density, c is the specific heat, $\vec{\mathbf{v}}$ is the fluid velocity and κ is the thermal conductivity. The parameter we want to preserve is the fluid speed. Thus the spatial discretization of eqn 1 is:

$$M_{cap}\dot{x} = (M_{cond} + v_s \cdot M_{conv})x + Load \cdot u \tag{2}$$

Here M_{cap} is the capacitance matrix, M_{cond} incorporates thermal conduction, M_{conv} contains the convection terms and $Load$ is the load vector. Thus all the information of the flow profile is contained in the convection matrix. Since convection is only applied to the elements in the flow channel this matrix does not have non-zero entries for every node and is thus not positive definite. The factor v_s is a scalar parameter. It can be used to scale the influence of the convection: the profile of the flow stays constant, but its magnitude can be scaled. With the assumption that the flow profile will not change for different absolute velocities, we can use it this model to generate a reduced-order-model where we can set the flow speed with a parameter.

1.3 The reduction process

To perform order reduction on the model it is necessary to export the disretized model from ANSYS. This is done with the GPL'd tool mor4ansys, which we use to read ANSYS binary files and to write the matrices as matrix-market files [5]. This tool can also perform model order reduction, however at the moment it is limited to the conventional approaches.

The further processing of the data is done within the computer algebra system Mathematica. This includes reading the matrices, performing the order reduction, solving the system and post-processing the result. Mathematica routines for these different tasks written at the IMTEK are collected in the IMTEK Mathematica Supplement (IMS) [6]. This software collection is also released under the GPL and contains - besides the already mentioned functions - many useful programs for engineering problems. Of course everyone is strongly encouraged to test the IMS and mor4ansys himself. The used reduction technique is described in the following.

2 Padé based Model Order Reduction

Many dynamic systems describing physical problems show a rapid decay of Hankel singular values. This property makes it possible to use Krylof subspace methods to generate a low order approximation of the original high dimensional model. These techniques are rather established in research to use with standard linear first order systems. The probably most often used algorithm is the Arnoldi process. As this algorithm is the basis for our parameter preserving approach it will be shortly described in the following.

2.1 The Arnoldi process

We start with a non-parametric system of very high order (eqn 3). E and A are quadratic matrices, the load B may be either a rectangular matrix in the case of multiple input functions, or a vector. The input function u is, respectively to B, either a vector or a scalar, x is the state vector.

$$E\dot{x} = Ax + Bu \tag{3}$$

In the Arnoldi process at first we define the transfer function $G(s)$ of the system in the frequency domain (eqn 4). Now a Taylor series expansion of this function is performed, where the first n moments of the expansion form a Krylov subspace. To ensure numerical stability this subspace is not used directly, but an orthonormal basis V is constructed that serves as a projection matrix to build the reduced model of order n (eqn 5).

$$\begin{aligned} G(s) &= -A^{-1}B(I - sA^{-1}E)^{-1} && (4)\\ E_r\dot{x}_r &= A_r x_r + B_r u && (5)\\ & \quad ,E_r = V^T E V, A_r = V^T A V \\ & \quad ,B_r = V^T B, x_r = V^T x \end{aligned}$$

The so generated model can then be computed like the original one. As it is of a much smaller order than the original model it can be computed with a fractional amount of the computational resources otherwise needed. To regain the complete high dimensional solution the solution of the reduced system just has to be projected back using V.

This method has already been applied to various problems out of different physical domains and is known to work reliably. Thus it can be very effectively used as a fast solver procedure, e.g. during a geometry optimization run. However, for automatic compact model generation in our case this algorithm is not so well fitted, as the only variables for such a model would be the input function.

2.2 Parametric model reduction

To enable parametric model reduction the differential equation system has to contain parameters. In the following one parameter will be used, however, the presented method is in not restricted to one, but can in principle handle any number of parameters. A comparable procedure was already proposed for geometrically parameterized interconnect modeling [4]. As our implementation aims at a wide range of mathematical systems we had to make minor changes to the algorithm which mainly affect the handling of the mixed modes.

In the case of one parameter the equation system 3 changes to eqn 6 with the transfer function eqn 7. It should be stated here that the parametric part can be in any matrix or even in different ones. For a mechanical system that means that it is possible to order-reduce a model with a parametric damping or/and stiffness matrix. For the presentation here we use the form our proposed model also has. Another important part is that only the matrix A_1 has to be invertible.

$$E\dot{x} = (A_1 + pA_2)x + Bu \quad (6)$$
$$G(s) = -A_1^{-1}B(I - sA_1^{-1}E + pA_1^{-1}A_2)^{-1} \quad (7)$$

The reduction process is in principle similar to the Arnoldi process. The main difference is that the Taylor expansion in not only performed around s, the complex frequency, but also around the parameters. This leads to pure moments for the frequency and the parameters, but also to mixed moments, where there are contributions from the frequency and from the parameters. These mixed moments can rapidly increase the dimension of the reduced model if they are all included into the projection matrix separately. This is especially the case, if the different matrices contain completely different physical effects, like e.g. convection and diffusion, as this increases the needed dimensions of the reduced models quite noticeably. Therefore in our implementation we decided to average the mixed moments of one iteration into one vector. Thus, in the case of one parameter, every iteration enlarges the projection matrix by 3 vectors; the pure moments for s and p and the combined mixed moments.

3 Results

The goal in model order reduction is to find a good approximation to a high-dimensional model by generating a low-dimensional model and thus saving resources. To judge the quality of the approximation computational results are compared between the reduced-order model and the original model. Of additional interest is of course the savings in resources that can be achieved using this technique.

The reduced model has a dimension of 102. Compared to non-parametric reduction, where reduced models often are of a smaller order then 50 this is rather big, however, it is obvious that adding parameters to the reduced model results in bigger models. Even more so, if the different matrices do not affect all the nodes equally, as in this case, where the convection matrix only has non-zero entries for the nodes residing in the channel. At last also the type of problem we want to model demands for extreme accuracy. The temperature difference between the heaters typically lies below 1‰of the temperature at the heater, therefore the absolute error has to be extremely small, to represent the devices behavior correctly.

3.1 Computational results

Figure 2 shows contour plots of the simulation result at different times. It is a visualization of the complete, high-dimensional solution vector where the time integration was performed with the low dimensional reduced model. To regain the complete state vector it is only necessary to make a reverse projection of the reduced state vector. This also shows the high potential of order-reduction techniques as fast-solver-procedures: performing a cheap time integration on the low-order model to gain the full, high-dimensional result.

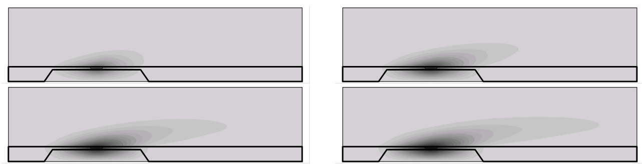

Figure 2: Transient results for $t_1 = 0.006s$, $t_2 = 0.012s$, $t_3 = 0.018s$, $t_4 = 0.024s$

While the contour plots are well suited to demonstrate the principle functionality of the procedure, they are not very convenient to determine the error induced by the order reduction. As the relevant output for an anemometer simulation is the difference temperature between the sensors, this value is also used to compare the solution of the complete model with the solution of the reduced model.

Figure 3 shows this temperature difference for steady state solutions with different velocities. The small graph inside the figure shows the deviation between the reduced model and the original model. This deviation lies below 1% of the difference signal and in the order of magnitude of 10^{-5} compared to the temperature at the nodes itself.

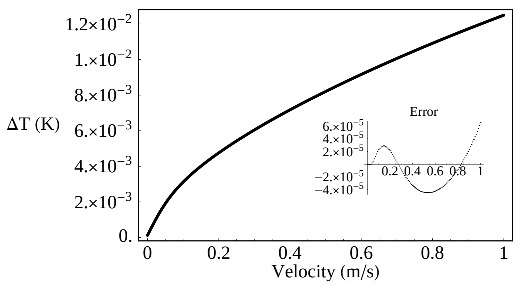

Figure 3: Steady state solution for the temperature difference between the sensors for different flow velocities

The reduced model fits the steady state solutions very well, however the more interesting test is to compare transient results, where the savings regarding the requirements on computational resources with the order-reduction approach are much bigger.

Figure 4 shows the transient response of the system to a step excitation. It can be seen that all the curves converge to the steady state solution. Commenting on the error it stands out that at the beginning a rather big error is done, while the error after $0.02s$ is very small again. The reason for this behavior is that the reduced model approximates the transfer function at a specific frequency that was chosen for the expansion. The Laplace transform of a step function however incorporates a wide range of frequencies. So when generating the model it has to be decided, whether it is more important to match the steady state, or if the high frequency part is more important. This trade-off can be minimized by using a multi-point expansion, where the same expansion is done for several frequencies, but this naturally further increases the dimension of the reduced model.

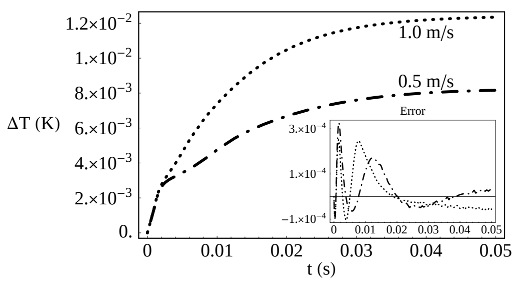

Figure 4: Transient results for $v = 0$, 0.5 and $1.0m/s$

Another application of the reduced model is shown in figure 5. Here the scalar parameter v is a time dependent specified variable, leading to a time-variant system. However, it is not possible to compare this result to results computed with ANSYS, as ANSYS can only handle constant convection terms.

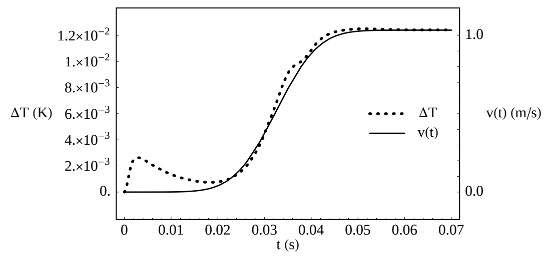

Figure 5: Temperature difference between right and left sensor for time-varying flow speed

3.2 Computational resources

For standard non-parametric reduction procedures the reduction resources typically lie in the range of the steady state solutions. This is not achievable in this case. However, the reduction process stays with 92s clearly below the 10 minutes ANSYS needs for the transient solution of the problem. However, once the reduction is performed the transient solution of the reduced model computes in less than one second. Due to the parametric approach we can thus perform the rather costly reduction process once and reuse the reduced model very effectively for with different parameter values.

4 Conclusion

We have shown that parameter preserving model order reduction techniques can be effectively applied to convection-diffusion problems. Even the demanding application of an anemometer can be handled with very good results. Thus it is a very good approach to automatic compact modeling and reduction of parametric or time-variant systems.

REFERENCES

[1] R.W. Miller, "Flow Measurement Engineering Handbook", McGraw-Hill, 1996

[2] www.ansys.com

[3] C.A.J. Fletcher, "Computational Techniques for Fluid Dynamics", Springer Verlag, 1991.

[4] L. Daniel, C.S. Ong, J. White, "A Multiparameter Moment-Matching Model-Reduction Approach for generating Geometrically Parameterized Interconnect Performance Models", IEEE Transactions on computer aided design of integrated circuits and systems, Vol. 23, 678-693, May 2004.

[5] www.imtek.de/simulation/mor4ansys

[6] www.imtek.de/simulation/mathematica/IMSweb