Two-way Coupled Structural Acoustic Optimization via Model Order Reduction (MOR).

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Abstract

A reduced order model is developed for low frequency, fully coupled, undamped and constantly damped structural acoustic analysis of interior cavities, backed by flexible structural systems. The reduced order model is obtained by applying a *Galerkin* projection of the coupled system matrices, from a higher dimensional subspace to a lower dimensional subspace, whilst preserving some essential properties of the coupled system. The basis vectors for projection are computed efficiently using the Arnoldi algorithm, which generates an orthogonal basis for the Krylov subspace containing moments of the original system. A computational test case is analyzed, and the computational gains and the accuracy compared with the direct method in ANSYS. Further, the reduced order modelling technique is applied to a two-way coupled vibro-acoustic optimization problem, with stacking sequences of the composite structure as design variables. The optimization is performed via a hybrid search strategy combining outputs from Latin Hypercube Sampling (LHS) and Mesh Adaptive Direct Search (MADS) algorithm. It is shown that reduced order modelling technique results in a very significant reduction in simulation time, while maintaining the desired accuracy of the optimization variables under investigation.

1 Introduction

Designing for quiet interiors is one of the key objectives during the product development cycle of a modern passenger vehicle or a commercial airplane. In order to gain competitive advantage, manufacturers are striving to reduce noise and vibration harshness (NVH) levels. As a result, design engineers often seek to evaluate the low frequency NVH behaviour of automotive/aircraft interiors using coupled finite element-finite element (FE/FE) or finite element-boundary element (FE/BE) discretized models. Due to the coupling between the fluid and structural domains in the coupled FE/FE formulation, the resulting mass and stiffness matrices are no longer symmetrical. In addition to this, as a general rule of thumb 10-15 linear elements are required per wavelength to get reasonable prediction accuracy for coupled structural acoustic problems. With wavelengths decreasing for increasing frequency, the model size drastically increases with frequency. This presents a major problem especially where optimization is required, with many design variables to be optimized. Therefore, generation of reduced order models, for fast coupled structural acoustic analysis and optimization is of great interest to the NVH community.

The two most popular approaches currently used to reduce the computational time of such coupled problems are the mode superposition and the component mode synthesis (CMS) method. The former method uses the dominant natural frequencies and mode shapes, extracted from a normal modal analysis, and the response is assumed to be a linear combination of the modes. In the later method, the system is divided into different components, and the frequency response is projected onto a fluid FE or a BE mesh to compute pressure levels. However, the reduction thus obtained is often not substantial. Further, the CMS method relies on the user to select interface nodes to enforce coupling conditions, which is a possible source of additional error. Other approaches to decrease computational time include generation of Ritz vectors, the use of influence co-efficients from a BE model, truncated coupled FE/FE analysis, and the patented ATV method, to name a few. The reader is referred to [1], for a review of some other approaches to reduce computational time. More recently, however, model order reduction (MOR) via implicit moment matching, has received considerable attention among mathematicians and the circuit simulation community [2, 3, 4]. It has been shown in various engineering applications [4, 5] that the time required to solve reduced order models via MOR is reduced significantly when compared to solving the original higher dimensional model, whilst maintaining the desired accuracy of the solution. The aim of MOR is to construct a reduced order model, from the original higher dimensional model, which is a good representation of the system input/output behavior at certain points in the frequency domain. The reduction is achieved by applying a projection from a higher order to a lower order space using a set of Krylov subspaces, generated by the Arnoldi algorithm. Additionally, the reduced model preserves certain essential properties such as maintaining the second order form and stability.

A reduced order model does not allow us to preserve geometry related information, and after changes in the original higher dimensional model, the reduced order model must be regenerated again. Fortunately, the time required to generate a reduced order model is comparable with that for a single frequency evaluation [11]. As a result, implicit moment matching is considered in this paper as a tool for fast frequency sweep. During the optimization, the reduced order model is used just once, and is regenerated for each optimization step. Nonetheless, we will demonstrate that even with such a set-up, there is considerable saving in computational time.

The paper focuses on the application of such Krylov based MOR techniques to structurally damped, fully coupled structural acoustic problems. The rest of the paper is laid out as follows: In Section- 2, the general framework for model order reduction for second order systems is introduced. In Section- 3 the Arnoldi procedure adapted for model order reduction for the coupled damped structural acoustic problem is described. In Section- 4 a numerical example from is solved using the direct approach in ANSYS FE code and the MOR via Arnoldi approach. Error estimates, results and computational times and simple convergence models are discussed. In Section- 5 MOR is incorporated via the Arnoldi process in the structural-acoustic optimization process to speed up simulation time, whilst maintaining the desired accuracy of the optimization variables and objective function under investigation. Section- 6 summarizes the paper with a short discussion of the results. Finally, Section: 7 concludes the paper with some potential applications of MOR in structural acoustics, and future recommendations.

2 Model Order Reduction for second order systems

After discretization of a general dynamical model of mechanical system, one obtains a system of second order ordinary differential equations in matrix form as follows:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Fu(t) \qquad (1)$$

$$y(t) = L^T x(t)$$

where *(t)* is the time variable, *x(t)* is the vector of state variables, *u(t)* is the input force vector, and *y(t)* the output measurement vector. The matrices *M*, *C* and *K* are mass, damping and stiffness matrices, *F* and

L are the input distribution matrix and output measurement matrix at certain points respectively. A harmonic simulation, assuming $\{F\} = F_0 e^{i\omega t}$ and ignoring damping in (1) yields:

$$[-\omega^2[M]+[K]]\,\{x\}=\{F\} \tag{2}$$
$$y(\omega)=L^T x(\omega)$$

where, ω denotes the circular frequency, and, $\{x\}$, $\{F\}$ denote complex vectors of state variables and inputs to the system respectively. The principle of model reduction is to find a lower dimensional subspace $V \in \Re^{Nxn}$, and,

$$x = Vz + \varepsilon \text{ where, } z \in \Re^n,\ n << N \tag{3}$$

such that the time dependent behaviour of the original higher dimensional state vector x can be well approximated by the projection matrix V in relation to a considerably reduced vector z of order n with the exception of a small error $\varepsilon \in \Re^N$. Once the projection matrix V is found, the original equation (2) is projected onto it. The projection produces a reduced set of system equations, in second order form, as follows:

$$(-\omega^2[M_r]+[K_r])\{z\}=\{F_r\} \tag{4}$$
$$y_r(\omega)=L_r{}^T z(\omega)$$

where the subscript r denotes the reduced matrix and:

$$M_r = V^T M V,\ K_r = V^T K V,\ F_r = V^T F,\ L_r = V^T L.$$

It is worth noting that $y_r(\omega) \approx y\ (\omega)$. Due to its low dimensionality, the solution to (4) is much faster than the original higher dimensional model. The input and output vectors are the same dimension as (2). Several methods exist to choose V. In this work, we choose the projection matrix V to be a Krylov subspace in order to provide the moment matching properties [2, 3].

2.1 Model Order Reduction for coupled structural acoustic systems:

For a coupled structural acoustic case, we start off from Cragg's pressure formulation [9]:

$$\begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix}+\begin{pmatrix} Cs & 0 \\ 0 & Ca \end{pmatrix}\begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix}+\begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix}=\begin{Bmatrix} Fs \\ 0 \end{Bmatrix} \tag{5}$$

$$y(t)=L^T\begin{Bmatrix} u \\ p \end{Bmatrix}$$

where, M_s is the structural mass matrix, M_{fs} is the coupled mass matrix, M_a is the acoustic mass matrix, C_s is the structural damping matrix, C_a is the acoustic damping matrix, K_s is the structural stiffness matrix, K_{fs} is the coupled stiffness matrix, K_a is the acoustic stiffness matrix, F_s is the structural force vector, $y\ (t)$ the output measurement vector and u, p are the displacements and pressures at nodal co-ordinates respectively. Ignoring damping for the structure and fluid, the coupled equations in the case of harmonic response analysis become:

$$\left[-\omega^2 \begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix} + \begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix} \right] \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} Fs \\ 0 \end{Bmatrix} \tag{6}$$

Constant structural or acoustic damping ratio's can be incorporated into the system matrices of (6) avoiding a direct participation of [C], as it is frequency independent by the definition. Although there exists techniques to reduce system matrices with [C], in this paper, we restrict ourselves to constant structural damping. A straightforward extension can be made to constant acoustic damping. The finite element software, ANSYS [14] formulates constant damping via the command DMPRAT and MP, DMPR which adds imaginary terms to the stiffness matrix according to the relationship:

$$\beta_c = \frac{2}{\Omega}\xi \tag{6-A}$$

Where, β_c is the constant multiplier applied to structural parts of the coupled stiffness matrix, and Ω is the frequency in rad/s and ξ is the constant damping ratio. This implies that the matrix [K]is complex-valued. In other words, the structural stiffness matrix K_s in Eqn (5), Eqn (6) becomes $K_s + 2\xi\ K_s$.

It can be seen that (6) is similar to (2). In this case, the approximation becomes:

$$\begin{Bmatrix} u \\ p \end{Bmatrix} = \{x\} = Vz + \varepsilon \tag{7}$$

The transfer function of the system H(s) = (Y(s) / $U(s)$) using the Laplace transform can be written as:

$$H(s) = L^T (s^2 M_{sa} + sC_{sa} + K_{sa})^{-1} F_{sa} \tag{8}$$

Ignoring damping, and expanding (8) using the Taylor series about $s = 0$ results in:

$$H(s) = \sum_{i=0}^{\infty} (-1)^i L^T (K_{sa}{}^{-1} M_{sa})^i K_{sa}{}^{-1} F_{sa} s^{2i} = \sum_{i=0}^{\infty} m_i s^{2i} \tag{9}$$

Where $m_i = (-1)^i L^T (K_{sa}{}^{-1} M_{sa})^i K_{sa}{}^{-1} F_{sa}$ $(for\, i = 0, \infty)$ are called the moments of $H(s)$ and,

$$M_{sa} = \begin{pmatrix} Ms & 0 \\ Mfs & Ma \end{pmatrix}, K_{sa} = \begin{pmatrix} Ks & Kfs \\ 0 & Ka \end{pmatrix}, F_{sa} = \begin{Bmatrix} Fs \\ 0 \end{Bmatrix}.$$

By matching some of these moments of the higher dimensional system about *s=0*, the reduced order model can be constructed, as it directly relates the input to the output of the system. Theoretically, any expansion point within the frequency range of interest can be used, and a real choice depends on required approximation properties. However, explicitly computing such moments tends to be numerically unstable [3, 4], and it is therefore preferable to attempt to implicitly match these moments via the Arnoldi process. Su and Craig [7], showed that if the projection matrix V is chosen from a Krylov subspace of dimension q,

$$\mathrm{K}_q(K^{-1}M, K^{-1}F) = span\{K^{-1}F, (K^{-1}M)K^{-1}F, \ldots\ldots\ldots\ldots (K^{-1}M)^{q-1}K^{-1}F\} \tag{10}$$

then, the reduced order model matches $q+1$ moments of the higher dimensional model. Loosely speaking, if the q^{th} vector spanning the Krylov sequence is present in matrix V, we match the q^{th} moment of the system. The block vectors $K^{-1}F$ and $K^{-1}M$ can be interpreted as the static deflection due to the force distribution F, and the static deflection produced by the inertia forces associated with the deflection $K^{-1}F$ respectively.

3 The Arnoldi Algorithm

To avoid numerical problems while building up the Krylov subspace, an orthogonal basis is constructed for the given subspace. This is done using the Arnoldi algorithm. Given a Krylov subspace $K_q(A_1, g_1)$, the Arnoldi algorithm finds a set of vectors with norm one which are orthogonal to each other, given by:

$$V^T V = I \quad \text{and} \quad V^T A_1 V = H_q \tag{11}$$

Where $H_q \in \Re^{qxq}$ is a block upper Hessenberg matrix and $I_q \in \Re^{qxq}$ is the identity matrix. *Figure: 1* describes the implemented algorithm, which is used to generate the Arnoldi vectors for the coupled structural acoustic system. For multiple inputs, the block version of the algorithm can be found in [4].

For the coupled structural acoustic case, we have:

$$Colspan(V) = K_q({K_{sa}}^{-1}M_{sa}, {K_{sa}}^{-1}F_{sa})$$

$$V^T {K_{sa}}^{-1} M_{sa} V = H_q \text{ and } V^T V = I \tag{12}$$

The discussion of the block version of the algorithm, which is used to generate the Arnoldi vectors for the coupled structural acoustic system , multiple inputs and multiple outputs, is quite involved, and so the reader is referred to [3] for a detailed discussion of this. In short, the block version of the Arnoldi algorithm generates orthogonal vectors spanning the Krylov subspace:

$$\mathrm{K}_q(A_1, g_1, g_2) = span\{g_1, g_2, A_1 g_1, A_1 g_2, \ldots\ldots\ldots\ldots A^{q-1} g_1, A^{q-1} g_2\} \tag{13}$$

It can be seen that in each step of the algorithm, one vector orthogonal to all previously generated vectors is constructed and normalized. The process is numerically very similar to the modified Gram-Schmidt orthogonalization. Due to the iterative property of the algorithm, it is possible to produce a reduced order model of lower dimension than initially specified, by just discarding the columns in matrix *V* and subsequently the rows and columns of the reduced order matrices. Eqn (6-A) implies that the matrix [K], and thus the Arnoldi generated projection matrix [V] are complex-valued.

Algorithm: 1:

Input: System Matrices K_{sa},M_{sa}, F_{sa}, L and *n* (number of vectors), expansion point $s = (\omega_E + \omega_B)/2$

Output: *n* Arnoldi vectors

0. Set $v_i = g$

1. For $i = 1 \rightarrow n, do:$

    1.1 Deflation check: $t_{i,i-1} = \| v_i \|$

    1.2 Normalization: $v_i = v_i^* / t_{i,i-1}$

    1.3 Generation of next vector: $v_{i+1}^* = Av_1$

    1.4 Orthogonalization with old vectors: for j=1 to i:

        1.4.1 $t_{j,1} = v_j^T v_{i+1}^*$

        1.4.2 $v_{j+1}^* = v_{i+1}^* - t_{j,i} v_j$

2. Discard resulting H_q, and project $M_{sa}, K_{sa}, F_{sa}, L$ onto V to obtain reduced system matrices $M_{Rsa}, K_{Rsa}, F_{Rsa}, L_{Rsa}$ where the subscript *Rsa* represents the reduced structural acoustic matrices.

Figure: 1: Arnoldi Process [3] [4].

4 Numerical Test Case

To evaluate the accuracy and the computational gains achievable via reduced order modeling, a numerical test case has been chosen. The test case is a constrained 1.5m x 0.8m damped fiber reinforced sandwich plate (Glass Fibre/Foam Core/ Glass Fibre) backed by a rigid walled acoustic cavity. A structural damping ratio of 4% is specified for the analysis. The frequency range of interest for the coupled dynamic analysis is 0-300Hz. A unit harmonic force is applied to one of the structural nodes to excite the coupled system as show in *Figure: 2(a)*. The pressure response is computed at three nodes in the fluid domain using the Direct Method in ANSYS and MOR via the Arnoldi process. All computations described in this paper were performed using a Pentium 3GHz, 2GB RAM machine.

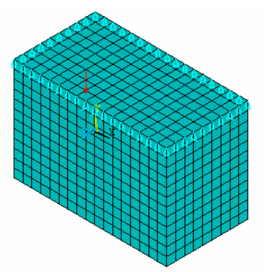

Figure 2 – (a): Coupled FE model

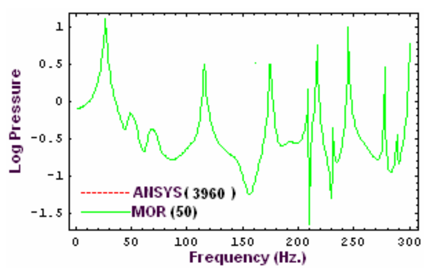

Figure 3 – (a): Noise Transfer Function

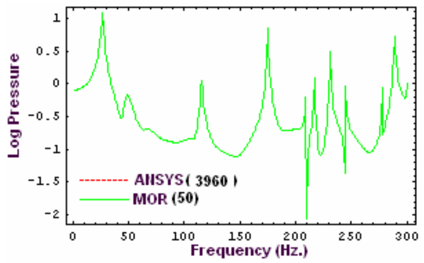

Figure 2 – (b) Bottom: Noise Transfer Function

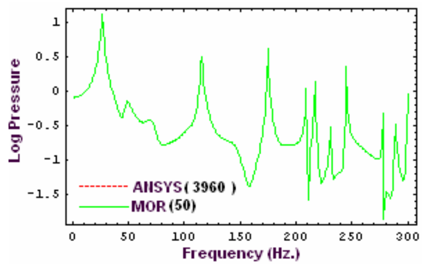

Figure 3 – (b) Bottom: Noise Transfer Function

For the reduced order model, the computational time is a combination of four steps (a) Running a Stationary solution and generating matrices (b) Reading matrices and generating of Arnoldi vectors (c) Projection to second order form and (d) Simulation of the reduced order model. The spilt computational times for test case 1 are given in *Table: 1*. The reduced order model is set up and solved in Mathematica/MATLAB environment. A comparison of the solution times using MOR and the Direct method in ANSYS are given in *Table: 2*.

Model	**ANSYS Stationary**	**Read Matrices , Arnoldi Vector Generation, projection**	**Reduced model Simulation**	**Total: MOR via Arnoldi**
TC[1]	4 s	43.1 s (50 Vectors)	1.25 s	48.35 s

Table: 1: MOR Split Computational Times; TC[1]: Test Case-1

Model	**Elements**	**DOF's**	**ANSYS Direct**	**MOR via Arnoldi**	**Reduction**
TC[1]	8400	11427	1080 s	48 s	96 %

Table: 2: Computational Times; TC[1]: Test Case-1

4.1 Convergence Properties

In this section, a method to compute the error estimates, and thus the convergence properties of the model is presented. The approach is similar to the method described in [10]. In the first convergence model, a straightforward *true error* between the two models is computed as:

$$\varepsilon_r(s) = \frac{\left| H(s) - H_r(s) \right|}{\left| H(s) \right|} \tag{15}$$

Where, $H(s)$ corresponds to the original transfer function, given by $H(s) = L^T(s^2 M_{sa} + K_{sa})^{-1} F_{sa}$ where, the definitions of M_{sa}, K_{sa} and F_{sa} remain the same as in (9) and $H_r(s)$ is the reduced order transfer function. Further, a *relative error* between two successive reduced order models r and $r+1$ can be defined as:

$$\hat{\varepsilon}_r(s) = \frac{\left| H_r(s) - H_{r+1}(s) \right|}{\left| H_r(s) \right|} \tag{16}$$

As discussed in *Section:3*, the iterative property of the algorithm, allows reduced order models of lower dimension than initially specified to be produced, by just discarding the columns in the matrix V and subsequently the rows and columns of the reduced matrices. The true and relative error plots are given in *Figure: 5* for start and end frequencies of the test cases i.e $\omega = 1\ Hz.$ and $\omega = 300\ Hz.$ *Figures: 5(a)* and *5(b)* indicate that, for the coupled box problem it is not possible to approximate the system using more than 50 Arnoldi generated vectors for both $\omega = 1\ Hz.$ and $\omega = 300\ Hz.$ The number of vectors required to adequately approximate the transfer function, depends on the model size and the number of inputs specified to excite particular modes of the system. Error plots based on the absolute values of the transfer function are shown in *Figure: 4(a)* and *4(b)* .It is worth noting that, the output does not participate in the reduction process, and so the approximation is completely independent of the number and location of outputs specified.

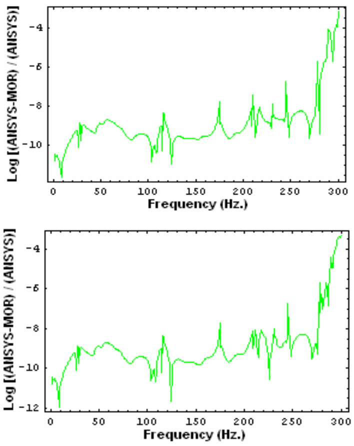

Figure: 4: (a) Top: Error Plot for 3(a)
(b) Bottom: Error Plot for 3(b)

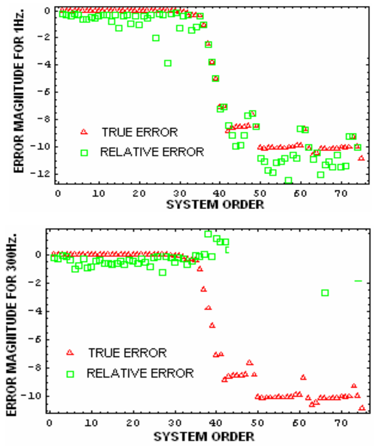

Figure: 5 (a): Top: Convergence model for 1Hz(b) Bottom Convergence model for300Hz

5 Coupled Vibro-Acoustic Optimization

In order to improve the acoustic characteristics of a vehicle interior, numerical optimization is often employed. Since there exists two forms of solution (coupled and uncoupled), it is often left to the engineer to decide on the approach best suited to the problem under investigation. However, a ‘one-way’ coupled analysis ignores the fluid loading on the structure, which is often the cause of cavity boom at low frequencies. Therefore, a fully coupled analysis is preferred in many vehicle/aerospace applications, but the computational time required to solve (6) restricts its subsequent use. Marburg [1] has provided a detailed review of the current practices in structural-acoustic optimization.

Over the recent years, different novel materials have been developed to control noise and vibration in vehicles and commercial aircrafts. In particular, fiber reinforced composites have generated significant interest in the development of structural materials due to their low density, high stiffness and excellent damping characteristics. Additionally, the orthotropic nature of such fiber reinforced composite materials implies that the directional stiffness depends on the orientation of fibers. Such flexibility can often be exploited to tailor the material to obtain the required structural acoustic performance [12], [13]. In this work, the feasibility of reducing interior noise levels through optimal lamination angles of a composite sandwich structure via reduced order modeling is demonstrated.

5.1 Optimization Test Case

The first test case is a frame-panel structure which was built to test new modelling techniques. The half scale car cabin is modeled as a simple seven sided structure as shown in *Figure: 6(a)*. The structural model is a frame panel structure coupled with the acoustic cavity. Faces of the acoustic model, other than that of the roof were assumed to be acoustically rigid. The acoustic model was modeled using eight noded acoustic brick elements with one pressure degree of freedom at each node. The panel is a laminated composite structure, made up of Fibre reinforced skins and a random fibre foam core. The cross section of the composite structure along with fiber orientation angles is shown in *Figure: 7*

The lamination angles of the fibres, denoted by θ in this study, are the design variables for the optimization problem. The design variables are subject to a lower and upper bounds $0 \le \theta \le 180$ degrees, where 0^0 represents a unidirectional lay-up of the fibers and angles beyond 90^0 represent lay-up in the negative direction. The structural FE model is modeled using ANSYS SHELL181 elements, with material properties for the uni-directional composite as given in *Table: 3*. For the cavity, a matching interface element (ANSYS FSI) is used, where the nodes of the structural and acoustic models are coincident. A structural damping ratio of 4% is applied to all elements with composite material properties. An initial lamination angle of 90/0/Core/0/90 is specified for the coupled analysis. The effects of changing lamination angles on the overall structural integrity of the structure are ignored. Similar to the previous test case, the frequency range of interest for the coupled dynamic analysis is 0-300Hz. The coupled model is excited by applying a unit harmonic force to one of the structural nodes of the front end lower member in the normal direction, and the response is computed at a node representative of driver's ear location in the fluid domain.

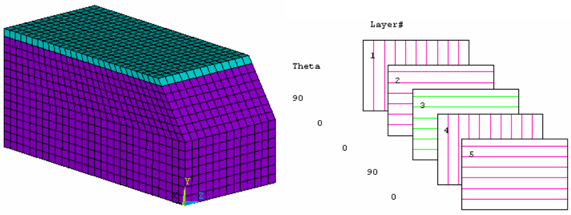

Figure: 6: (a-left) Top: Coupled FE model (b-right) Stacking sequence of fibers

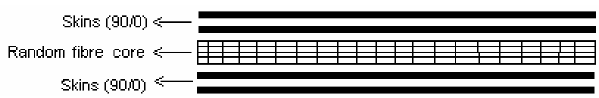

Figure: 7: Cross Section of sandwich composite

Cross Section	E11= E33 (GPa)	E22 (GPa)	G12=G21 (GPa)	G13 (GPa)	v12=v13	Density (kg/m^3)	Lamination Angles (^{0})
Skin	28	21	1.39	1.40	0.4	1480	90/0/Core/0/90
Core	9.26	4.2	0.72	0.67	0.4	800	--

Table: 3: Mechanical properties for Fibre reinforced sandwich composite.

5.1.1 Design Optimization Procedure

The reduced order modeling technique outlined in *Section: 2* is incorporated into the optimization process to speed up simulation time, while maintaining the accuracy of the nodal sound pressure values. A general framework of optimization via reduced order modeling is given in *Figure: 10*. The number of vectors required to represent the higher dimensional system is calculated using the convergence models presented in *Section: 4.1*. In this case, 50 Arnoldi generated vectors were sufficient to represent the higher dimensional system for $\omega = 1\ Hz.$ and $\omega = 300\ Hz.$ An error plot over the entire range of frequencies is shown in *Figure: 8*. The true and relative error plots at 1Hz. for the optimization problem are shown in *Figure: 9*. Since it is the lamination angles which this study is seeking to optimize, and thus the material properties of the composite structure, the reduced order model must be regenerated at each iteration involving a change in the lamination angle.

Mathematically speaking, the non-linear optimization problem can be stated as:

Find a vector of design variables: $\theta = (\theta_1, \theta_2, \theta_3, ..., \theta_n)$

Which minimizes the objective function $f(\theta)$,

Subject to lower and upper bounds $\theta_i^{lower} \leq \theta_i \leq \theta_i^{upper}$

Numerous possibilities exist to formulate the objective function, and their effectiveness depends on the nature of the coupled problem. For the optimization problem stated above, the objective function is formulated as:

$$f(\theta) = \hat{F}^{\frac{1}{n}} \quad ; \quad \hat{F} = \frac{1}{(\omega_{\max} - \omega_{\min})} \int_{\omega_{\min}}^{\omega_{\max}} \vartheta \{p_i(\omega)\}\, d\omega \tag{17}$$

$$\vartheta = \begin{cases} (p_i - p_{ref})^n & for \quad p_i > p_{ref} \\ 0 & for \quad p_i \leq p_{ref} \end{cases} \tag{18}$$

Where, the function ϑ is a weighting function applied to the nodal sound pressure level (SPL) value p_i. It can be seen that the weighting function depends on reference pressure p_{ref}, determined as 90dB for the current study.

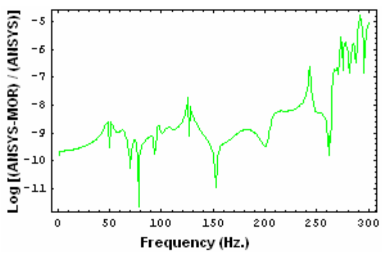

Figure 8 –Error plot for optimization test case

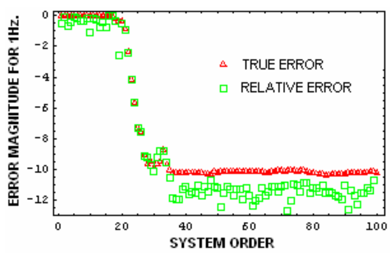

Figure 9 – True, Relative Error Plot for 1Hz.

At this point, any value of p_{ref} can be used, and in this case is chosen to reduce peak SPL values over the entire frequency range of 0-300Hz. For *n=2*, this formulation of objective function (17, 18) results in a frequency averaged root mean square value. This ensures that the higher peaks of the noise transfer function are given more importance, avoiding deep valleys as compensations for high peaks during the optimization process In particular, the fluid resonance peaks at ~120Hz, ~265Hz, and ~293Hz (See *Figure: 15*) have been identified as target zones where sound pressure values are to be decreased.

The optimization is carried out using MATLAB GA/PS Toolbox [17] using the Mesh Adaptive Direct Search (MADS) algorithm. MADS is a class of derivative free algorithms, specifically designed for non-smooth optimization problems, and is an extension of the generalized pattern search (GPS) algorithm. A pseudo code for GPS algorithm is shown in *Figure: 11*. Each iteration of MADS is divided into two steps, SEARCH and POLL. The SEARCH step allows the evaluation of the objective function at a finite set of

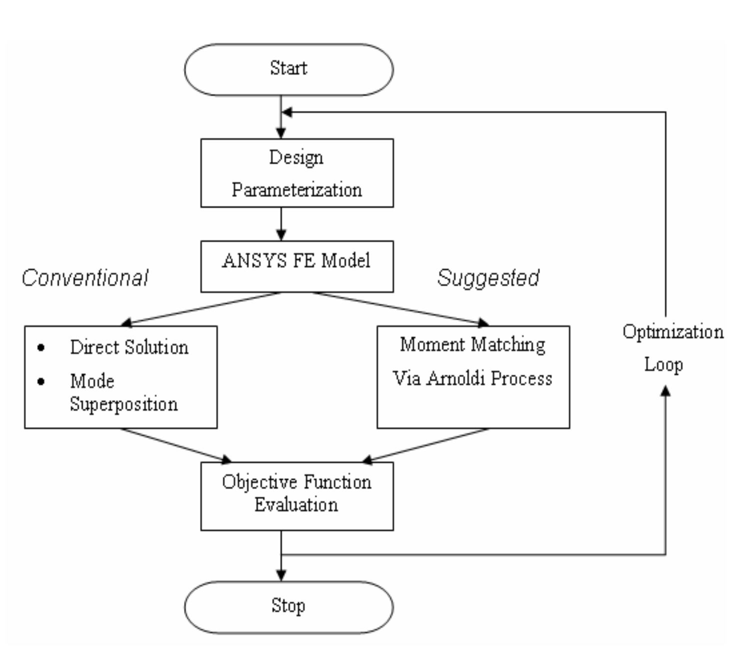

Figure: 10: Optimization via Moment Matching.

k=1

Currentfitness = fitness(meshloc)

While $\Delta_k \geq 1E-6$

Bestfitness = currentfitness

for i = 1… n

if odd(i) then dir=+1, else dir=-1 endif

newloc=meshloc

trialfitness=fitness(newloc)

if trialfitness < best fitness

Bestloc=newloc

$\Delta_{k+1} = 2(\Delta_1)$

else $\Delta_{k+1} = 0.5(\Delta_k)$ *endif* $k = k+1$

endwhile

Figure: 11: Simplified Pattern Search algorithm.

points. Any search strategy can be used, including none. When a SEARCH step fails to improve the objective function value, a POLL step is invoked. The key difference between GPS and MADS lies in this POLL step. In addition to the mesh size parameter, a poll size parameter is defined to ensure that the local exploration of the design variable space is not restricted to a finite set of directions [15]. The set of trial points considered during the POLL step is called a *frame*. Depending on the result of the POLL step, i.e. successful or unsuccessful, the mesh resolution is decreased or increased .A general MADS algorithm is shown in *Figure: 12*. In this work, we chose to evaluate initial trial points in the first iteration of MADS using Latin Hypercube Sampling (LHS).

Algorithm:2:

INITIALIZATION: Define mesh point, mesh size and poll size parameters, set *k* ← *0*.

SEARCH AND POLL STEP: Perform SEARCH and POLL steps until an improved mesh point is found on mesh.

- OPTIONAL SEARCH: Evaluate function on a finite subset of trial points on mesh.
- LOCAL POLL: Evaluate function on computed *frame*.

PARAMETER UPDATE: Update mesh size and poll size, set *k* ← *k+1* and return to SEARCH and POLL steps.

Figure: 12: A simplified MADS Algorithm [14].

5.2 Results

A general decrease in SPL values is apparent over the entire frequency range of optimization. Results at specific frequencies for a node representative of drivers ear location is summarized in *Table: 5*. It can be seen that the composite material is no longer symmetrical in terms of lamination angles after optimization. From an initial lay up of 90/0/Core/0/90 the lamination angles move towards a lay up of 66/154/Core/141/144. *Figure: 15* compares the sound pressure level before and after optimization. The face sheets of the outer layer of the composite material tend to be moving towards a more cross-ply orientation (66/-26) while the inner lamination angles moves towards an even -39/-36. Sound pressure levels near fluid resonant frequencies ~120Hz, ~265Hz, and ~293Hz have decreased by 1.2dB, 4.75dB and 6.1dB respectively. However, in the frequency range of 150-240Hz., peak SPL value has increased from 83.9dB to 85.2dB – an increase of 1.4dB. This is primarily because (a) The optimization is considered over the entire frequency range of 0-300Hz., and an overall decrease in the root mean square SPL value is considered as a successful iteration by the optimizer and (b) the value of $p_{ref} = 90dB$ is applied to (18). In addition to this, the structurally damped resonant peak causing fluid excitation at 200Hz. has moved to 180Hz. This result can be attributed to *shifting* of modes during the optimization process. Such a shift in noise-emitting modes is due to the change in stiffness of the structure, as an effect of a change in lamination angles of the composite.

Model	Function Evaluations	Initial Stacking Sequence	Final Stacking Sequence	Time MOR	Time ANSYS*	Time Reduction
OPT[1]	156	90/0/C/0/90	63.75/154/C/141/144	36816s	982800s	97%

*Table: 4: Optimization Results: OPT[1]: Optimization Test Case.*Estimated time.*

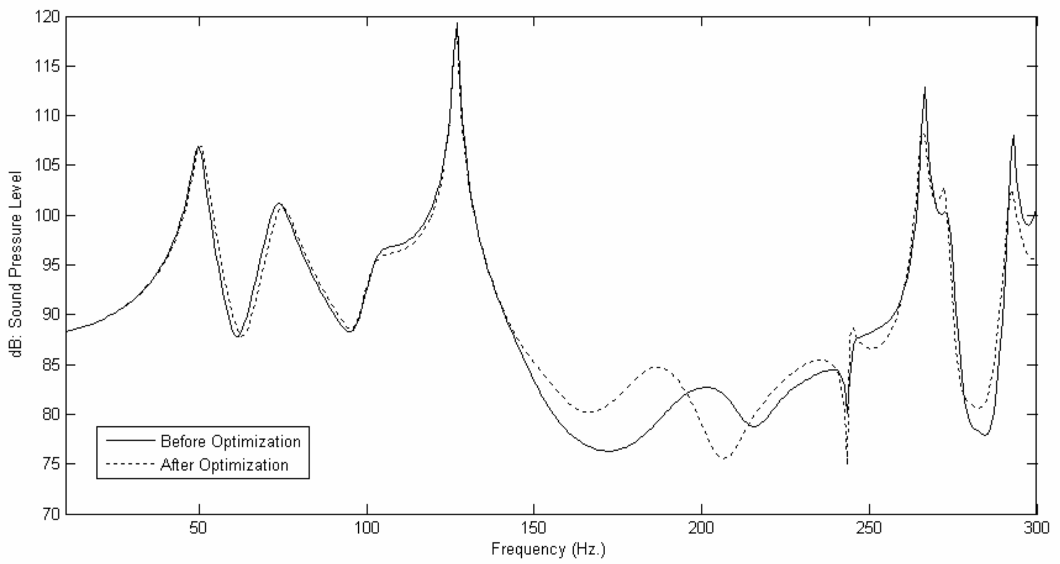

Figure: 15: Noise Transfer Function before and after optimization.

From a computational viewpoint, the use of reduced order modeling significantly decreases the computational time requirements for the optimization process. The time taken by MOR via Arnoldi is shown in *Table: 4*. For this case, the cost of reduction via Arnoldi is approximately 97% smaller than the original higher dimensional ANSYS model. Such smaller cost enables effective application of hybrid search strategies, to search the multi-modal space which usually requires more number of function evaluations. If the optimization had been carried out by the direct method in ANSYS, 400 hours of computational time would have been required for 156 function evaluations. To compare the accuracy of the optimized design variables, a harmonic simulation via direct method in ANSYS is performed. Noise transfer function at the selected fluid node for optimization is shown in *Figure: 14*. It can be seen that there is an almost perfect match between ANSYS and reduction via Arnoldi process.

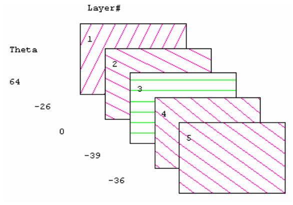

Figure: 13 : Optimized Stacking Sequence

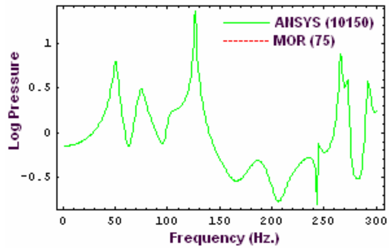

Figure: 14: Noise Transfer Function for optimized design variables.

Frequency (Hz.)	SPL (dB) (Before Optimization)	MOR-Arnoldi SPL (dB) (After Optimization)	ANSYS SPL (dB) (66/154/Core/141/144)
70	98.41	96.42	96.42
120	100.50	99.21	99.21
200	82.65	79.20	79.20
265	112.81	108.06	108.06
293	108	101.87	101.87

Table: 5: Optimization Results

6 Summary

An efficient method to perform coupled structural analysis and optimization via reduced order modelling has been outlined. The basis vectors for matching the coupled system moments are computed by applying the Arnoldi algorithm, which computes the projection vectors spanning the Krylov subspace, to match the maximum number of moments of the system. The moments in the test cases shown are matched at approximately half of the analysis range $s = (\omega_E + \omega_B)/2$. If a Taylor series expansion is considered around a higher frequency, a reduced order model could be obtained with better approximation properties around that frequency range. *Figure: 2(b), 3(a), 3(b), 4(a), 4(b), 5(a)* and *5(b)* indicates that good approximation properties can be obtained by projecting the higher dimensional system to a lower dimension and matching some of the low frequency moments of the system. While there exist several methods to choose basis vectors, we have chosen these vectors to span the Krylov subspace. Compared to the computing eigen modes and eigen vectors of the system matrices, computing vectors spanning the Krylov subspace is much faster and efficient, since a normal modal analysis of a complex structural or an acoustic system tends to be computationally expensive. In fact, there is no guarantee that the computed modes included for the mode superposition via a modal analysis would be enough for the time/harmonic analysis, and often an approximate guess of modes within the $2n$ range are computed for projection, n being end frequency [8]. Moreover, for the fully coupled structural acoustic case, an unsymmetric solver is required to extract coupled modes, which leads to further inefficiency in the calculation procedure. *Figure: 2(b)*, *3(a)* and *3(b)* show that the reduced order model accurately captures the dynamic behavior of the coupled higher dimensional system, indicated by peaks at ~114.50Hz and ~172Hz. and ~215Hz, which correspond to the acoustic modes of the cavity.

Although there are many techniques to generate the projection matrix [V], in this work we have chosen the Arnoldi process to generate vectors belonging to the Krylov subspace. Another popular method to generate the projection vectors belonging to the Krylov subspace is the Lanczos process. *Table-6* summarizes the advantages and disadvantages of Arnoldi over Lanczos. When compared to the Lanczos process, the Arnoldi process is more stable, and a complete approximation of the output is guaranteed by the Arnoldi process. This means that, for the coupled structural acoustic case, both displacements on the structural domain, and pressure levels at fluid nodes could be matched. Although this has not been verified explicitly in either the test case or the optimization problem, existing literatures show that a complete match is specific to the Arnoldi process (e.g. [9]). In [19], this has been numerically demonstrated for a Electro-Thermal model. Further, it is possible to partition the projection matrix, thus preserving the displacement and pressure state variables. For the first test case, 25 outputs were chosen for the analysis, which included both normal displacements on the structural portion of the model and pressure levels in the fluid domain. The number of vectors needed to accurately represent the system was 50 and 75 for test case 1 and the optimization test case respectively. The difference in the number of vectors needed can be attributed to the nature of coupled models itself and its resulting transfer function. It is also worth noting that the process of computing the minimum number of required vectors can be completely automated by a user defined error parameter. Lastly, the reduced order modelling framework via the Arnoldi process was incorporated in the structural-acoustic optimization process. The lamination angles of the composite structure took the form of design variables for the optimization problem. As stated earlier, any change in material properties required generation of reduced order model from the higher dimensional model. A general decrease in SPL over the entire frequency range of optimization is evident. Both *mode shifting* and *peak splitting* phenomenon's resulting from change in lamination angles were accurately captured by the reduced order model. It should be noted that a reference value of $p_{ref} = 90dB$ was specified for the optimization. A different value of p_{ref} could result in different optimized stacking sequence for the composite. However, such a parametric study is beyond the scope of this paper. The error plots shown in *Figure: 4(a), 4(b), 8* indicate that the difference between the higher dimensional ANSYS model and reduced order model is almost negligible. It can be observed that the error increases with increasing frequency.

PROPERTY DESCRIPTION	**ARNOLDI PROCESS**	**LANCZOS PROCESS**
Accuracy of approximation	*r* moments of the system match	*2r* moments of the system match
Computational Complexity	$O(2r^2n + 2rN_z(K_{sa}{}^{-1}M_{sa})$	$O(16rn + 4rN_z(K_{sa}{}^{-1}M_{sa})$
Complete output approximation	YES	NO
Numerical stability of algorithm	YES	NO
Preserving stability and passivity of the reduced order model	YES	NO

Table: 6: Comparison between model order reduction via Arnoldi and Lanczos processes.

This result can be primarily attributed to the choice of expansion points used to generate the reduced order model. A reduced order model could have been obtained, by matching moments at different expansion points, with each expansion point requiring a separate factorization (leading to rational Krylov methods). However, for the optimization problem discussed in this paper, the maximum error for the specified fluid node is in the order of 10^{-5}. The convergence plot for the optimization problem indicates that it is not possible to approximate the system with more than 50 Arnoldi generated vectors. At this point, machine precision is reached, and a further better approximation cannot be found. For the optimization test case, MOR via Arnoldi results in a 97% reduction in simulation time. The reduction seems consistent with different test cases.

7 Conclusion

A new method to develop efficient reduced order models for fully coupled structural acoustic analysis and optimization has been outlined. The basis vectors for model reduction are computed by applying the Arnoldi algorithm, which computes the projection vectors spanning the Krylov subspace, to match the maximum number of moments of the system. The method would serve as an excellent alternative to many other reduction techniques, particularly for vibro-acoustic optimization, where reduction of computational time is often sought. In addition to this, a complete output approximation is guaranteed, matching both displacements of the structure and sound pressure levels in the fluid. In this paper, the explicit participation of [C] is avoided by using a complex stiffness approach $K_s\ (1+2\xi)$. For a higher dimensional model with [C], a reduced order model via the Second Order Arnoldi (SOAR) process, involving computing orthogonal vectors belonging to the second order Krylov subspaces [20] [21] or transforming the *Equation (6)* to first order form and matching moments via Arnoldi is possible, but a comparison of accuracy and efficiency of such methods is beyond the scope of this current paper. Finally, it should be noted that MOR via Arnoldi or Lanczos would be appropriate only in the low frequency range for vehicle/aerospace structural acoustic applications, and other techniques exists (e.g. EFEA, SEA) to deal with higher frequencies, where modal density is often high, and the acoustic response is very sensitive to minor structural modifications.

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