ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

A COMPARISON OF STRUCTURAL-ACOUSTIC COUPLED REDUCED ORDER MODELS (ROMS): MODAL COUPLING AND IMPLICIT MOMENT MATCHING VIA ARNOLDI

R Srinivasan Puri, Denise Morrey, John F. Durodola

School of Technology, Oxford Brookes University, Oxford OX33 1HX, United Kingdom

Rick C. Morgans, Carl Q. Howard

School of Mechanical Engineering, University of Adelaide, SA, Australia 5005

s.puri@brookes.ac.uk
rick.morgans@adelaide.edu.au

Abstract

The calculation of the coupled response of a structure with an enclosed acoustic cavity is of great interest, with many practical applications in the automotive and aerospace industries. Traditional methods such as a fully coupled fluid-structure interaction finite element calculation can be very computationally expensive, and methods have been proposed that reduce this computational burden and make it possible to include iteration and optimisation in the design process. This paper compares the performance of two such reduced order model (ROM) methods, a traditional modal coupling technique and an implicit moment matching method via Arnoldi, with a fully coupled finite element calculation. A simple model, a square simply supported steel plate backed by a rigid walled cavity is used as an example, and the accuracy of each method is examined for both damped and undamped cases. It was found that Arnoldi gave excellent agreement with the fully coupled calculation, and that while modal coupling gave excellent agreement near resonance, the performance off resonance was dependent on the number of modes retained.

1. INTRODUCTION

The coupling between a structure and an internal acoustic cavity is a problem in many industries and in particular, it can cause Noise Vibration and Harshness (NVH) issues in road [1] and aerospace vehicles, even to the extent of damaging expensive satellites during launch [2]. The problem is becoming worse as the structural mass is reduced in an effort to improve the efficiency of transport systems.

Designers often use a finite element calculation of the coupled system to predict the sound levels found in the acoustic cavity. A finite element model involving structural and acoustic elements is formulated, with coupling between the structural and pressure degrees of freedom, resulting in unsymmetric stiffness and mass matrices. This large matrix is then subsequently solved at each frequency in order to predict the response of the system. This process, called the fully coupled finite element method, can be very computationally intensive and hence unsuitable for inclusion in iterative design processes such as design optimization.

A number of techniques have been proposed to speed up the solution of this method [1]. The well known modal coupling technique [3–6], presented by Fahy [7] is one such method. This uses the *in vacuo* modal responses of a structure and hard walled acoustic modal response of a cavity and combines them into a coupled vibro-acoustic response. The advantage of using modal-coupling theory is that the computation time is significantly reduced, even though the numerical results are almost identical.

More recently, however, model order reduction (MOR) via implicit moment matching has received considerable attention among mathematicians and the circuit simulation community [8, 9]. In this approach, a reduced order model is generated by matching the higher dimensional system *moments* via *Krylov subspace* based projection techniques. It has been shown in various engineering applications [10–13] that the time required to solve reduced order models via MOR is reduced significantly when compared to solving the original higher dimensional model, whilst maintaining the desired accuracy of the solution.

This paper presents results from a collaboration between The University of Adelaide, who have experience with modal coupling [14], and Oxford Brookes University, who have applied Arnoldi based reduced order modeling techniques to fully coupled structural-acoustic analysis and optimization problems[15]. This initial work presents results from both reduced order modeling techniques, applied to a very simple numerical model, for both damped and undamped cases. The rest of the paper is laid out as follows. Section 2 outlines Cragg's fully coupled FE/FE displacement/pressure formulation and briefly describes the theory behind two different reduced order modeling (ROM) approaches, the uncoupled modal coupling approach and implicit moment matching via the Arnoldi process. In section 3 a simple numerical example is chosen and solved using the three different approaches outlined in section 2. Section 4 concludes the paper with a general discussion of numerical results focusing on the accuracy of the ROM approaches, and highlights potential directions for future research.

2. THEORY

2.1. Fully Coupled FE/FE Formulation

Starting with the well known combined displacement-pressure (*u/p*) formulation for the structural-acoustic model as a whole [16]

$$\underbrace{\begin{bmatrix} M_s & 0 \\ M_{fs} & M_a \end{bmatrix}}_{M_{sa}} \begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \underbrace{\begin{bmatrix} C_s & 0 \\ 0 & C_a \end{bmatrix}}_{C_{sa}} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \underbrace{\begin{bmatrix} K_s & K_{fs} \\ 0 & K_a \end{bmatrix}}_{K_{sa}} \begin{Bmatrix} u \\ p \end{Bmatrix} = \underbrace{\begin{Bmatrix} F_s \\ F_a \end{Bmatrix}}_{F_{sa}} \tag{1a}$$

$$y(t) = L^T \begin{Bmatrix} u \\ p \end{Bmatrix} \tag{1b}$$

where M_s is the structural mass matrix, M_a is the acoustic mass matrix, K_s is the structural stiffness matrix, K_a is the acoustic stiffness matrix, M_{fs} is the coupling mass matrix, and K_{fs}

is the coupling stiffness matrix, C_s is the structural damping matrix, C_a is the acoustic damping matrix, u denotes the structural displacements, p denotes the nodal pressures in the fluid domain, and F_s and F_a denote the force(s) on the structural domain, or constrained acoustic pressure degrees of freedom (DOFs) and purely acoustic excitation, in the form of volume acceleration belonging to the fluid domain respectively. K_{sa}, M_{sa}, C_{sa} are the fully coupled structural-acoustic matrices of order $N \times N$. F_{sa} denotes the structural, acoustic excitation and is of an order $N \times m$, with m being the number of inputs to the system (for single input, $m = 1$). $y(t)$ is the output measurement vector and the matrix L^T is the output scattering or the so called *field point* matrix, which is an identity matrix, of order $N \times N$, in the case of a complete output of states (which in this case are displacements and pressures) being required.

Although there exist several techniques to reduce system matrices with [C], in this paper, we restrict ourselves to constant structural damping. The finite element software ANSYS formulates constant damping via the commands DMPRAT and MP, DMPR which adds imaginary terms to the stiffness matrix according to the relationship [17] $\beta_c = 2\zeta/\Omega$ where β_c is the constant multiplier applied to the structural parts of the coupled stiffness matrix Ω is the frequency in rad/s and ζ is the constant damping ratio.

In the *direct method*, the global matrices belonging to Eq. (1a) are assembled and the set of linear equations solved (in the frequency domain) at all frequencies of interest. Typically, sparse direct solvers [17] are employed to perform a LU decomposition at each frequency, which are then subsequently used to compute the desired values of states of Eq. (1a).

2.2. Modal coupling

Fahy [7, p249] describes equations for the coupled structural-acoustic response of a system in terms of the summation of structural and acoustic mode shapes. The structural displacement is described in terms of a summation over the *in vacuo* normal modes as

$$w(\mathbf{r}_s) = \sum_{p=1}^{\infty} w_p \, \phi_p(\mathbf{r}_s) \tag{2}$$

where ϕ_p is the mode shape of the p^{th} structural mode, $\mathbf{r}_s$ is an arbitrary location on the surface of the structure, and w_p is the modal participation factor of the p^{th} mode. Note that the time dependent term $e^{j\omega t}$ has been removed from this equation and others in the paper to simplify the analysis.

The acoustic pressure is described in terms of a summation of the acoustic modes of the fluid volume with rigid boundaries as

$$p(\mathbf{r}) = \sum_{n=0}^{\infty} p_n \, \psi_n(\mathbf{r}) \tag{3}$$

where ψ_n is the acoustic mode shape of the n^{th} mode, $\mathbf{r}$ is an arbitrary location within the volume of fluid, and p_n is the modal participation factor of the n^{th} mode. Note that the $n = 0$ mode is the acoustic bulk compression mode of the cavity that must be included in the summation.

The equation for the coupled response of the structure is given by [7, Eq. (6.27)]

$$\ddot{w}_p + \omega_p^2\, w_p = \frac{S}{\Lambda_p} \sum_n p_n C_{np} + \frac{F_p}{\Lambda_p} \tag{4}$$

where ω_p are the structural resonance frequencies, Λ_p are the modal masses, F_p are the modal forces applied to the structure, S is the surface area of the structure, and C_{np} is the dimensionless coupling coefficient given by the integral of the product of the structural (ϕ_p) and acoustic (ψ_n) mode shape functions over the surface of the structure, given by

$$C_{np} = \frac{1}{S} \int_S \psi_n(\mathbf{r}_s)\, \phi_p(\mathbf{r}_s)\, dS \tag{5}$$

The equation for the coupled response of the fluid is given by [7, Eq. (6.28)]

$$\ddot{p}_n + \omega_n^2\, p_n = -\left(\frac{\rho_0 c^2 S}{\Lambda_n}\right) \sum_p \ddot{w}_p C_{np} + \left(\frac{\rho_0 c^2}{\Lambda_n}\right) \dot{Q}_n \tag{6}$$

where ω_n are the resonance frequencies of the cavity, ρ_0 is the density of the fluid, c is the speed of sound in the fluid, Λ_n is the modal volume, and Q_n is the source strength with units of volume velocity (hence $\dot{Q}_n$ has units of volume acceleration).

The *in vacuo* structural and hard walled acoustic modes are calculated using a finite element package, and N_s structural and N_a acoustics modes are retained.

The equations for the fully coupled vibro-acoustic system, can be formed into a matrix equation using Equations (4), (5), and (6) as

$$\begin{bmatrix} \mathbf{A} & -S\,\mathbf{C} \\ -S\,\omega^2\,\mathbf{C}^\mathrm{T} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{w}_p \\ \mathbf{p}_n \end{bmatrix} = \begin{bmatrix} \mathbf{F}_p \\ \mathbf{Q}_n \end{bmatrix} \tag{7}$$

where **A** is a $(N_\mathrm{s} \times N_\mathrm{s})$ diagonal matrix with elements $A_{pp} = \Lambda_p(\omega_p^2 - \omega^2)$ and **B** is a $(N_\mathrm{a} \times N_\mathrm{a})$ diagonal matrix with elements $B_{nn} = \Lambda_n/\rho_0 c^2\,(\omega_n^2 - \omega^2)$. The off diagonal elements account for the cross coupling between structure and fluid, where **C** is a $(N_\mathrm{s} \times N_\mathrm{a})$ matrix with individual entries given by the elements of C_{np}. This matrix can be solved by matrix inversion techniques to find the coupled modal participation factors, and hence the coupled response of both the structure and cavity. Damping can easily be added on a modal basis [7, 14].

2.3. Model Order Reduction

Ignoring damping and rewriting Eqs. (1a,1b) using Laplace transforms, in terms of the input $U(s)$ and output $Y(s)$ which are related by the *transfer function* $H(s) = [Y(s)/U(s)]$, gives

$$H(s) = L^T(s^2 M_{sa} + K_{sa})^{-1} F_{sa} \tag{8}$$

Expanding Equation (8) using the Taylor series about $s = 0$ results in

$$H(s) = L^T(s^2 K_{sa}^{-1} M_{sa} + I)^{-1} K_{sa}^{-1} F_{sa} = \sum_{i=0}^{\infty} m_i s^{2i} \tag{9}$$

where, $m_i = (-1)^i L^T (K_{sa}^{-1} M_{sa})^i K_{sa}^{-1} F_{sa}$ are called the moments of $H(s)$. By matching some of these moments about $s = 0$, the reduced order model can be constructed, as it directly relates the input to the output of the system. Therefore, a reduced order model is now sought, which matches the higher dimensional coupled system moments. This can be achieved by choosing q basis vectors stored in a column matrix $[V]$ (of order $N \times q$), and then applying a subsequent Galerkin projection [18] on the higher dimensional system matrix (of order $N \times N$) leads to a reduced order model (of order $q \times q$), which for the coupled structural-acoustic case the approximation can be expressed as

$$\left\{ \begin{array}{c} u \\ p \end{array} \right\} = \left\{ \; x \; \right\} \approx Vz + \varepsilon \tag{10}$$

where $\{x\}$ contains the pressures and displacements to be approximated, and a small error parameter ε resulting from the projection to generalized co-ordinates. This form is often denoted as the *change of state co-ordinates*. In this work, we choose vectors for projection belonging to the Krylov subspace in order to provide the moment matching property [19]. Given a matrix $[A]$ and a vector g, a Krylov subspace of order q is defined by

$$K_q(A, g) = span(g, Ag, ...A^{q-1}g) \tag{11}$$

2.3.1. The Arnoldi Algorithm

To ensure numerical stability while building up the Krylov subspace $K_q(A, g)$ and that an orthogonal basis is constructed for the given subspace, the Arnoldi algorithm is used in this work to generate *candidate vectors* containing moments of the coupled higher dimensional system matrices. Given a Krylov subspace, the Arnoldi algorithm finds a set of vectors with norm one, that are orthogonal to each other, given by [8, 9, 18]

$$V^T V = I \; and \; V^T A V = H_q \tag{12}$$

where $[H_q]$ is a block upper Hessenberg matrix, and is an orthogonal projection of $[A]$ onto the Krylov subspace defined in Eq. (11). Figure 1 gives the simplified single-input, single-output/complete output (SISO/SICO) version of the implemented Arnoldi algorithm. For the fully-coupled structural-acoustic problem described, we have (*for* $s = 0$)

$$A = K_{sa}^{-1} M_{sa}, \; g = K_{sa}^{-1} F_{sa} \;\; ; \;\; V^T K_{sa}^{-1} M_{sa} V = H_q \; and \; V^T V = I \tag{13}$$

The initial dimension of q is chosen such that the input-output behaviour of the coupled system is well represented. In this case, since only q moments are matched, the approximation is said to be a Padé-type approximant. Once the projection matrix $[V]$ is found, $[H_q]$ is discarded and a Galerkin projection $\prod = [V][V]^T$ on undamped Eqs. (1a,1b) subject to harmonic excitation generates a reduced order model in second order form, given by

$$[-\omega^2 [M_{rsa}] + [K_{rsa}]]\{z\} = \{F_{rsa}\} \tag{14a}$$

$$y_{rsa}(\omega) = L_{rsa}^T z(\omega) \tag{14b}$$

Input: System Matrices K_{sa},M_{sa},F_{sa},L^T and q (Number of vectors)
and expansion point s, in this case $s = (\omega_e + \omega_b)/2$
Output: q Arnoldi vectors belonging to the Krylov Subspace
0. Set $v_1^* = g$
1. $for\ i = 1 \rightarrow q\ \ do:$
1.1 Deflation Check: $h_{i,i-1} = \|v_i\|$
1.2 Normalization: $v_i = v_i^*/h_{i,i-1}$
1.3 Generation of next vector: $v_{i+1}^* = Av_i$
1.4 Orthogonalization with old vectors $for\ j = 1 \rightarrow i\ \ do:$
1.4.1 $h_{j,i} = v_j^T v_{i+1}^*$
1.4.2 $v_{i+1}^* = v_{i+1}^* - h_{j,i}v_j$
2. Discard resulting H_q and project M_{sa},K_{sa},F_{sa},L^T onto V to obtain
reduced system matrices $[M_{rsa}]$,$[K_{rsa}]$,$\{F_{rsa}\}$,L_{rsa}^T

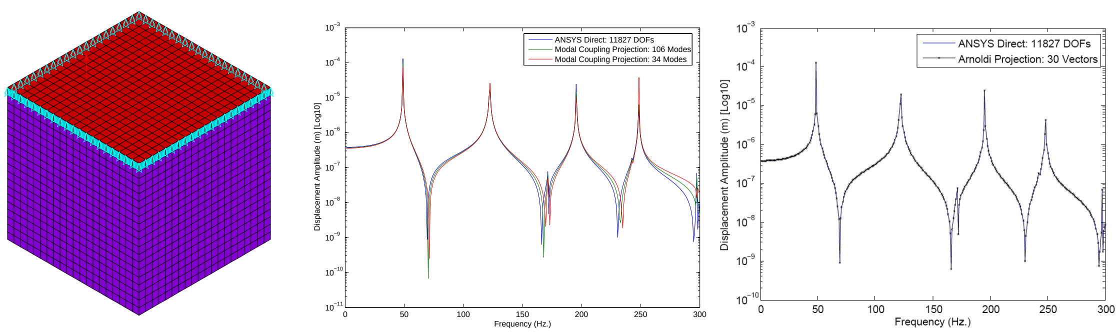
Figure 1. Complete SISO/SICO Arnoldi process [8, 9]

where $[M_{rsa}] = V^T M_{sa} V;\ [K_{rsa}] = V^T K_{sa} V;\ \{F_{rsa}\} = V^T\{F_{sa}\};\ L_{rsa}^T = L^T V.$

The dimension reduction from $N \times N \longrightarrow q \times q$ (where $q \ll N$) is now achieved, and the harmonic simulation of Eqs. (14a,14b) is much faster than Eqs. (1a,1b).

3. NUMERICAL TEST CASE

The test structure is a 1m $\times$ 1m steel plate constrained at the edges normal to the plate (in other words *simply supported*), backed by a rigid walled cavity of dimensions 1m $\times$ 1m $\times$ 1m. The harmonic analysis of the coupled equations were solved using three approaches: (a) the direct method using the ANSYS FE solver, which in-turn employs the LU decomposition method for all defined substeps; (b) MOR via the SISO/SICO Arnoldi algorithm; and (c) Modal Coupling (Full and Truncated). Two different structural damping values were used for the analysis (a) ζ = 0 and (b) ζ = 0.03, and no acoustic damping was included. A constant amplitude force excitation of 1N, over the frequency range from 0 to 300Hz, was applied at one of the off-centre nodes (0.25m,1m,0.65m) of the structural FE mesh as shown in Figure (2a). A total of 8400 elements were used for the coupled FE model.



(a) (b) (c)

Figure 2. (a) Fully coupled structural-acoustic FE/FE model, (b) ANSYS and Modal Coupling predictions for various numbers of retained modes and (c) ANSYS and Arnoldi predictions for 30 vectors.

In the moment matching approach, 30 basis vectors for matching the coupled system moments are computed by applying the Arnoldi algorithm described in the previous section.

For the modal coupling approach, the uncoupled modal responses are calculated using Finite Element Analysis, with structural and acoustic eigenvalue decompositions calculated sequentially. Modes were extracted to frequency 1.5 times higher that the maximum frequency considered in the analysis, giving a total of 34 retained modes. These modal responses are used in the modal-coupling theory to calculate the coupled vibro-acoustic response of the system.

The structural receptance transfer function (normal displacement over input structural force) at the driving point (0.25m,1m,0.65m) and a noise transfer function (cavity pressure over input structural force) at positions (0.5m,0.5m,0.5m) and (0.75m,0.75m,0.25m) inside the box were specified as outputs for the analysis. The results for the undamped and damped cases are presented in Figure 3.

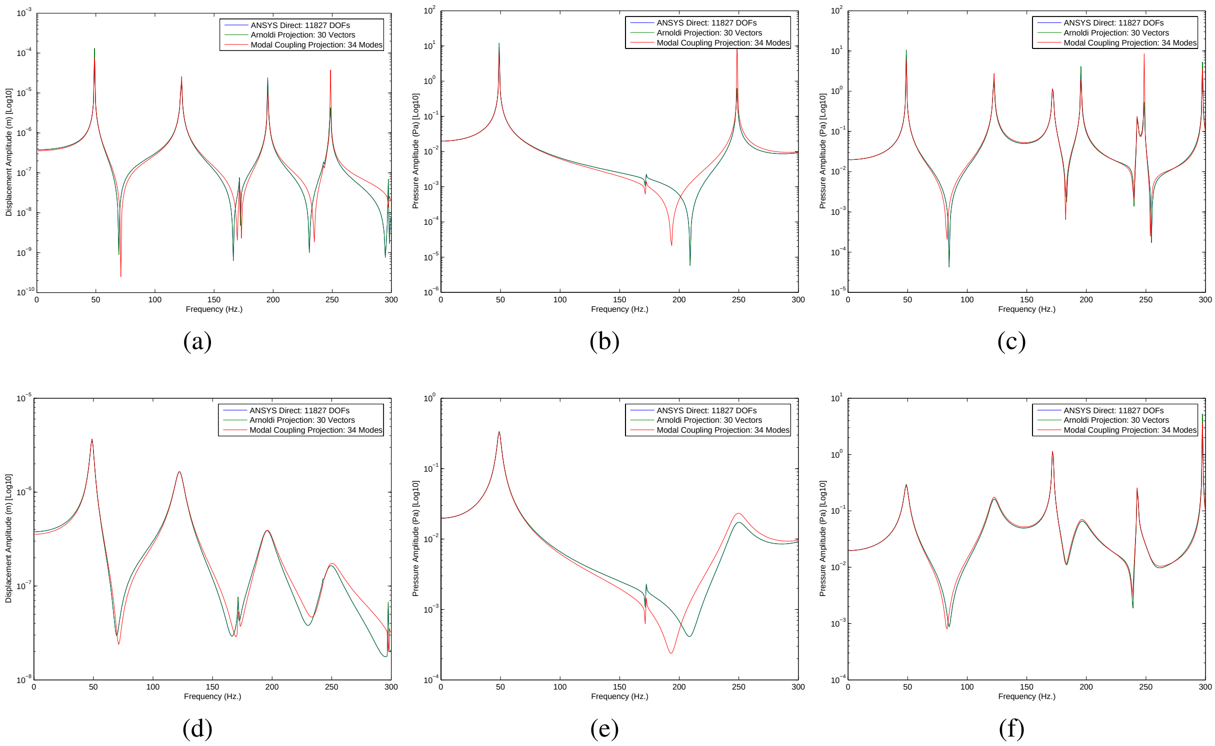


Figure 3. (a,b,c) Undamped (ζ=0), (d,e,f) Damped (ζ=0.03) ANSYS, Arnoldi and Modal Coupling predictions (a,d) point displacement FRF, (b,e) pressure FRF at (0.5m,0.5m,0.5m) and (c,f) pressure FRF at (0.75m,0.75m,0.25m).

4. DISCUSSION AND CONCLUSION

Figures 2c and 3 indicate that the reduced order model generated via the moment matching leads to excellent accuracy over the entire frequency range. The moments in the test case shown are matched at approximately half of the analysis range. If a Taylor series expansion is considered around a higher frequency, a reduced order model could be obtained with better approximation properties around that frequency range. The modal coupling approach gives good accuracy at resonance and poor accuracy off resonance, compared with the *direct method*. This phenomena is due to residues from truncated modes and is well documented [7]. For many applications, the contribution of the prediction off resonance may be minor (*e.g.* active control [14]). The effect of including more modes in the Modal Coupling method can be seen in Figure 2b. The accuracy of the prediction is improved, but it is still does not compare to the accuracy of the Arnoldi approach.

The current work focused on the accuracy of the ROM approaches. In terms of future work, generating ROMs with different loading conditions (*e.g.* acoustic excitation) would lead to a better understanding of the advantages and drawbacks of different ROM approaches. A comparison between total computational times is worth pursuing in the future. An accuracy comparison of derived secondary field variables (e.g. fluid velocity) from the primary field variable via different ROM approaches would further enhance the clarity of situations pertaining to the application of such reduced order models for fully coupled structural acoustic analysis.

REFERENCES

[1] S. Marburg. Developments in structural-acoustic optimization for passive noise control. *Archives of the computational methods in engineering*, 9(4):291–370, 2002. Ref: AA094A.

[2] B. Henderson, C. Gerhart, S. Lane, E. Jensen, S. Griffin, and A. Lazzaro. Vibro-acoustic launch protection experiment valpe. *Journal of the Acoustical Society of America*, 114(4):2384, 2003.

[3] R.H. Lyon and G. Maidanik. Power flow between linearly coupled oscillators. *Journal of the Acoustical Society of America*, 34:623, 1962.

[4] F.J. Fahy. Vibration of containing structures by sound in the contained fluid. *Journal of Sound and Vibration*, 10(3):490–512, 1969.

[5] L.D. Pope. On the transmission of sound through finite closed shells: Statistical energy analysis, modal coupling, and non-resonant transmission. *Journal of the Acoustical Society of America*, 50(3):1004–1018, 1971.

[6] E.H. Dowell, G.F. Gorman III, and D.A. Smith. Acoustoelasticity : General theory, acoustic natural modes and forced response to sinusoidal excitation, including comparisons with experiment. *Journal of Sound and Vibration*, 52(4):519–542, 1977.

[7] F. Fahy. *Sound and structural vibration: radiation, transmission, and response*. Academic Press, London, 1st edition, 1985.

[8] R. W. Freund. Krylov subspace methods for reduced order modeling in circuit simulation. *Journal of Applied Mathematics*, 123:395–421, 2000.

[9] Z. J Bai. Krylov subspace techniques for reduced order modeling of large scale dynamical systems. *Applied Numerical Mathematics*, 43:9–44, 2002.

[10] E.J. Grimme. *Krylov Projection Methods for Model Reduction*. PhD thesis, Dept. of Electrical Engineering, University of Illinois at Urbana Champaign, 1997.

[11] K. Willcox. *Reduced-Order Aerodynamic Models for Aeroelastic Control of Turbomachines*. PhD thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, 2000.

[12] G. Lassaux. High-fidelity reduced-order aerodynamic models: Application to active control of engine inlets. Master's thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, 2002.

[13] T. Bechtold, E. B. Rudnyi, and J. G. Korvink. Dynamic electro-thermal simulations of microsystem-a review. *Journal of Micromechanics and Microengineering, Institute of Physics publications*, 15:R17–R31, 2005.

[14] B.S. Cazzolato. *Sensing systems for active control of sound transmission into cavities*. PhD thesis, The University of Adelaide, March 1999.

[15] R.S. Puri, D. Morrey, J.F. Durodola, A. Bell, E.B. Rudnyi, and J.G. Korvink. Krylov subspace techniques for low frequency structural-acoustic analysis and optimization. *In Proceedings of Inter-Noise, USA*, v1, 2006.

[16] A. Craggs. The transient response of a coupled plate-acoustic system using plate and acoustic finite elements. *Journal of Sound and Vibration*, 15:509–528, 1971.

[17] ANSYS Inc. *ANSYS V10 Theory Manual*. 2005.

[18] A. C Antoulas. *Approximation of large-scale dynamical systems*. Society for Industrial and Applied Mathematics, ISBN: 0-89871-529-6, 1st edition, 2003.

[19] T. J. Su and R.R. Craig. Krylov model reduction algorithm for undamped structural dynamics systems. *AIAA Journal of Guidance and Control Dynamics*, 14:1311–1313, 1991.