Modern Model Order Reduction for Industrial Applications

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Dynamic analysis is an important part of modern simulation strategy. However, finite element models are inherently high dimensional and, as a result, their dynamic analysis is computationally expensive. For this reason, it is impossible to include a finite element model directly into system level simulation where a device dynamic model should be simulated as a part of a whole system including driving circuitry.

In structural mechanics, model reduction based on mode superposition or Guyan reduction has been in use for long time in order to speed up dynamic simulation [1]. These two techniques can be found nowadays in almost any commercial implementation of the finite element method and they are employed not only for structural mechanics but also for other physical domains. Model reduction can also be considered as a formal mathematical problem to approximate dynamic behavior of a high-dimensional model. Recently mathematicians extensively researched the problem from such a viewpoint [2]. They say that if we look at a model reduction problem formally then neither mode superposition nor Guyan reduction produces an optimal reduced model.

The goal of this paper is to introduce modern formal model reduction methods and then present examples of their use for different applications in design and system level simulation (see Fig. 1). We limit ourselves by model reduction for linear models as only in this case the model reduction process can be fully automated. We start by short introduction to linear model reduction. Then we discuss how it can be employed for the second order systems, as mathematicians developed the model reduction theory originally for the first order dynamic systems. After that

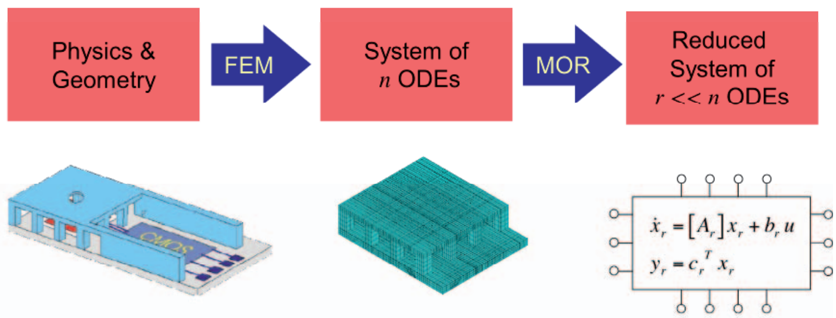

Figure 1: Model reductin is an efficient means to enable system-level simulation.

Method	Advantages	Disadvantages
SVD-based (Truncated Balanced Approximation, Singular Perturbation Approximation, Hankel-Norm Approximation).	Have a global error estimate, can be used in a fully automatic manner.	Computational complexity of conventional implementations is $O(N^3)$, can be used for systems with order less than a few thousand unknowns only.
Low-rank Gramian approximants (SVD-Krylov).	Have a global error estimate and the computational complexity is acceptable.	Currently under development.
Pade approximants via Krylov subspaces by means of either the Arnoldi or Lanczos process (implicit moment matching).	Very advantageous computationally, can be applied to very high-dimensional linear systems.	Does not have a global error estimate.

Table 1: Methods for linear model reduction (2)

we present the software MOR for ANSYS developed at IMTEK that can perform modern model reduction directly for finite element models made in ANSYS. Finally, we describe several industrial applications in which MOR for ANSYS has been employed.

Linear Model Order Reduction

After the discretization in space, we obtain a system of ordinary differential equations (ODEs). We start by a system of ODEs of the first order in the form accepted in the control theory.

$$E\dot{x} + Kx = Bu$$
$$y = Cx$$

where $\boldsymbol{x}$ is the state vector containing degrees of freedom in the finite element model, $\boldsymbol{E}$ and $\boldsymbol{K}$ are the system matrices. The main difference from a typical finite element notation is 1) splitting of the load vector to a product of a constant input matrix $\boldsymbol{B}$ and a vector of input functions $\boldsymbol{u}$ and 2) the introduction of the output vector $\boldsymbol{y}$ that contains some linear combinations of the state vector that are of interest in system level simulation.

The main assumption for model reduction is that the high dimensional state vector actually moves in the low-dimensional subspace and we can project the original system on that subspace (see Fig 2). As such, the goal of model reduction technique is to find the low-dimensional subspace $\boldsymbol{V}$ that accurately captures the dynamics of the state vector.

Input and output in dynamic system (1) affect its dynamic behavior considerably and it is important to take them into account during model reduction. Here is the main difference between modern model reduction with mode superposition and the Gyuan reduction. In order to find the low-dimensional subspace, mode superposition and the Guyan reduction use only the two system matrices, while modern model reduction uses all four matrices in Eq (1). Yet, it should be stressed that input functions $\boldsymbol{u}$ do not take part in the model reduction process and they are transferred from the original to the reduced model without any changes.

Antoulas [2] has suggested a classification of model reduction methods shown in Table 1. From a theoretical viewpoint, the best are the Singular Values Decomposition (SVD)-based methods, as they have global error estimates. As a result, model reduction can be made completely automatic. An engineer has to specify permissible tolerance for the approximation and then the required dimension of a reduced model is chosen based on the global error estimates. Unfortunately, computational time in this case grows cubically with the dimension of the state vector. SVD-based methods with better scaling computational properties are still under development and, at the moment, implicit moment matching is the only option that can be employed for industrial applications right now.

The idea of moment matching is to transform the dynamic system (1) into the Laplace domain and then to find such a low-dimensional system that has the same first derivates in the Taylor expansion around some point as the original model. The direct implementation of this idea is numerically very unstable but mathematicians have found that, by means of the generation of a particular Krylov subspace, one finds such a projection subspace that the reduced model certainly matches first moments. It is interesting to note that the use of moment matching for model reduction in structural mechanics can be traced back to works of Wilson [3] in 1982 and Craig [4] in 1991. Note that implicit moment matching is applicable to a linear dynamic system with both symmetric and unsymmetric system matrices.

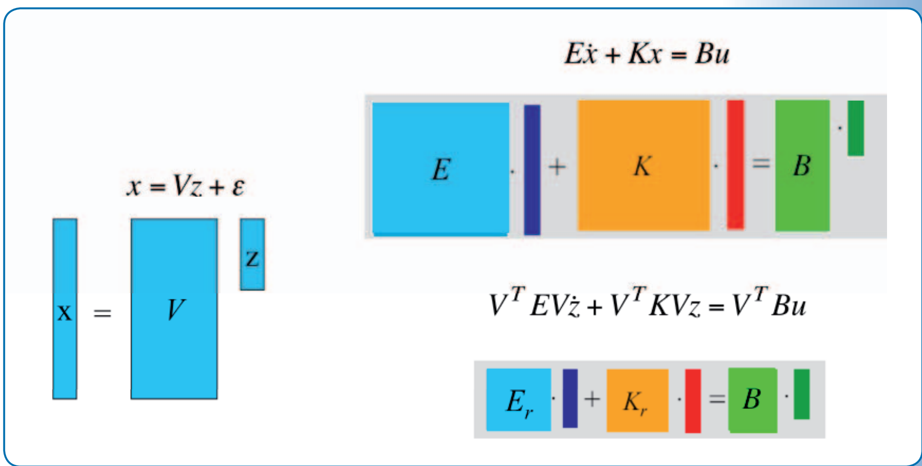

Figure 2: Model reduction as a projection of the high dimensional system onto the low-dimensional subspace.

Treatment of Second Order Systems

Most often, the discretization by the finite element method produces a system of ODEs of the second order as follows

$$M\ddot{x} + E\dot{x} + Kx = Bu$$
$$y = Cx$$

There are three options to proceed with model reduction in this case, as shown in Fig. 3. First, in the common case of proportional damping

$$E = \alpha M + \beta K$$

the damping matrix can simply be ignored during the process of constructing the projection basis. In this case, only the mass and stiffness matrices together with the input matrix are employed to generate the required Krylov subspace. The damping matrix is projected afterwards and because of (3) it can actually be computed from reduced mass and stiffness matrices. It is worthy to note that in the case of proportional damping, moment matching properties have been proved to hold for any values of **α** and **β** [5].

In the general case of non-proportional damping, it is always possible to transform dynamic system (2) to the first order system by increasing the dimension of the state vector twice. The disadvantage here is that a reduced system is obtained in the form of the first order system and that computational requirements increase because of the increase in the dimension of the state vector. There are new results from mathematicians following [4] that allow us to build a second order Krylov subspace. This removes both disadvantages mentioned before.

MOR for ANSYS

MOR for ANSYS (http://ModelReduction.com/) is open-source software developed originally at IMTEK to employ modern model reduction directly for ANSYS models [6]. It reads binary ANSYS FULL and EMAT files in order to extract the system matrices and then runs a model reduction algorithm (see Fig. 4). MOR for ANSYS uses implicit moment matching based on the block Arnoldi algorithm, as it is the most efficient computational method. Time to perform model reduction in this case is comparable with that for a static solution provided there is enough memory. In Table 2, the time to generate the low-dimensional subspace of dimension 30 by the Arnoldi algorithm is compared with the time for a static solution in ANSYS for several thermal and structural models. In our experience, 4 Gb of RAM is enough to treat this way many finite element models up to 500 000 degrees of freedom.

The structural model of butterfly microgyroscope (see Fig. 5) has been developed in ANSYS [7] and the number of active degrees of freedom after the discretization was 17361. MOR for ANSYS has been used to generate a reduced model of dimension 30. The harmonic

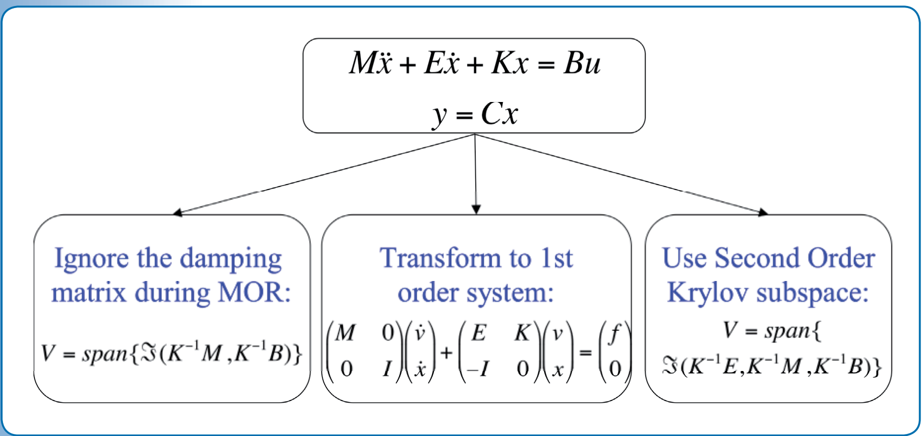

Figure 3:
Model reduction options for a second order system.

Dimension	Number of nonzeros in K	Static solution in ANSYS in s	Time to generate *V* in s
4 267	20 861	0.63	0.90
11 445	93 781	2.2	4.0
20 360	265 113	15	26
79 171	2 215 638	230	310
152 943	5 887 290	95	211
180 597	7 004 750	150	280
375 801	15 039 875	490	830

Table 2: Computational times on Sun Ultra-80 with 4 Gb of RAM

response as well as the relative difference between the original and reduced model in frequency range from 300 to 300000 Hz is shown in Fig. 6. The relative difference is about 10-9 up to 105 Hz, in other words, the reduced model describes the dynamic behavior of the original model in a wide frequency range with exceptional accuracy. The relative error increases to 1% after 105 Hz, yet it should be mentioned that the expansion point was equal to zero.

In the case when several simulations with different input functions are necessary (system-level simulation), the advantage of model reduction is out of the question. Yet, during the design phase, a reduced model should be generated each time when a user changes the geometry or material properties of the original model. In this case, a reduced model may be used just once. Nevertheless in the case of the Arnoldi algorithm, the model reduction time is considerably smaller than time for transient or harmonic response simulation of the original system. Hence, model reduction can also be used as a fast solver to speed up transient and harmonic response simulations. These two different situations for the use of model reduction are shown in Fig. 7.

Applications

In this section, we list several applications with references to the original papers where the use of linear model reduction implements the ideas displayed in Fig. 1 and Fig. 7. More case studies can be found at the MOR for ANSYS site.

A microhotplate device is

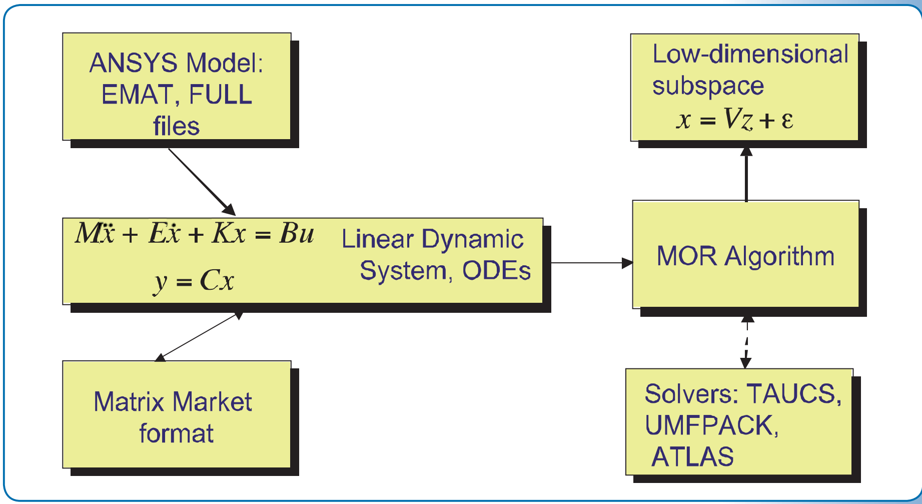

Figure 4:
MOR for ANSYS block scheme

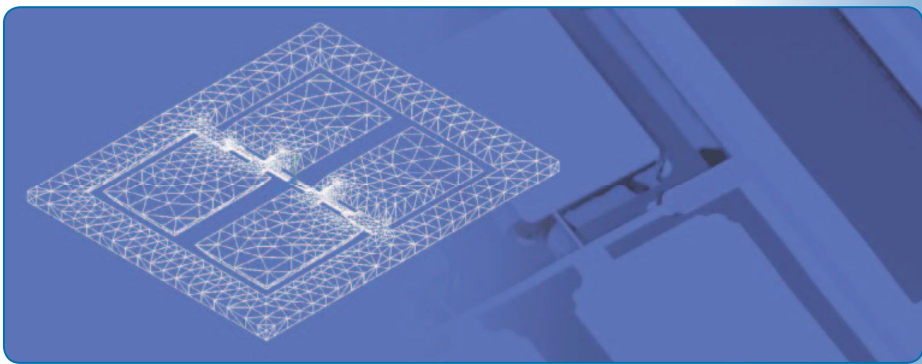

Figure 5:
The butterfly gyroscope model.

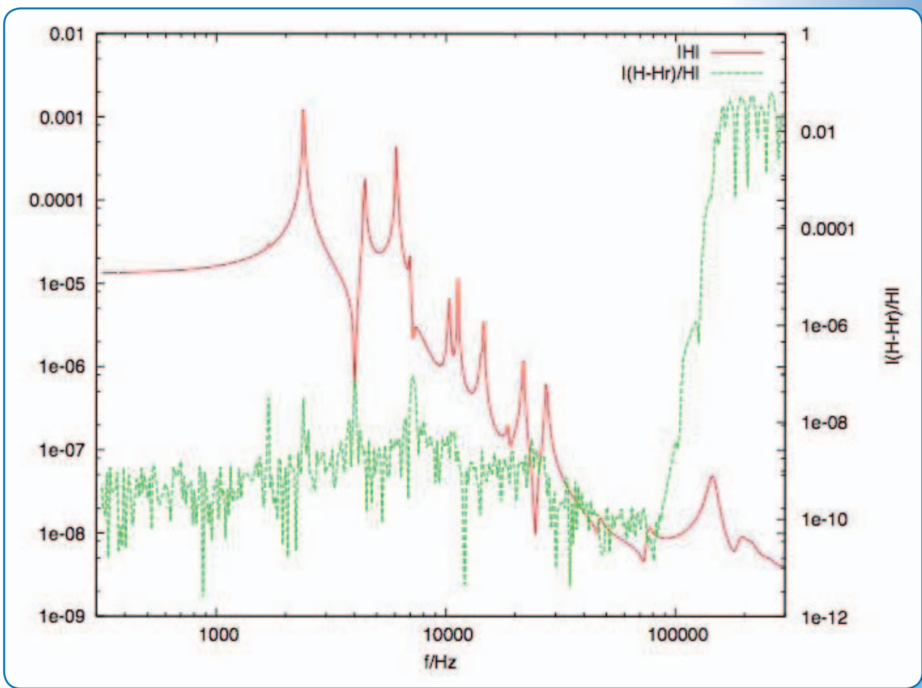

Figure 6:
Harmonic response simulation for the butterfly gyroscope model (red line). The green line shows the relative difference between the full and reduced models (the axis is on the right).

basically a resistor heated by current in order to keep the temperature at the desired level. Microhotplates are often employed in gas sensors, anemometers (flow meters), optical filters and other applications. Electro-thermal modeling is an essential part of the development. The assumption of homogeneous heat generation allows us to move the electrical part into the input function for a thermal part and thus makes possible the use of linear model reduction. The methodology of using model reduction in this case is presented in [8]. It should be mentioned that this problem was the starting point for the MOR for ANSYS development.

The response of a microaccelerometer is dynamic by its nature, as the inertia effects cannot be neglected. Hence the performance function to optimize the microaccelerometer must include results of transient simulation. In Ref [9], the optimization of a microaccelerometer has been performed with the use of MOR for ANSYS in order to speed up transient simulation and hence reduce optimization time.

Radio frequency microelectromechanical systems (RF MEMS) are nowadays considered to be a main building block of future generations of reconfigurable wireless terminals. During the design phase of RF-resonators, their electromechanical harmonic response has to be accurately predicted and eventually modified in order to satisfy with desired specifications. This requires a harmonic pre-stressed analysis, since the harmonic signal is generally superimposed to a static voltage, which influences the device electrical and mechanical properties (a small-signal approximation). In Ref [10], a strategy has been developed to use MOR for ANSYS as a tool to automatically extract a compact model of an RF-resonator for circuit optimization directly from the finite element model.

Computational acoustics including fluid-structure interaction is most often based on the linear approximation and, as such, it is well suited for model reduction. For example, in a modern passenger vehicle or a commercial airplane, the noise, vibration and harshness (NVH) performance is one of the key parameters, which the customer uses to assess product quality. To this end, acoustics simulation is indispensable to evaluate the low frequency NVH behavior of automotive/aircraft interiors during the design phase. In Ref [11],

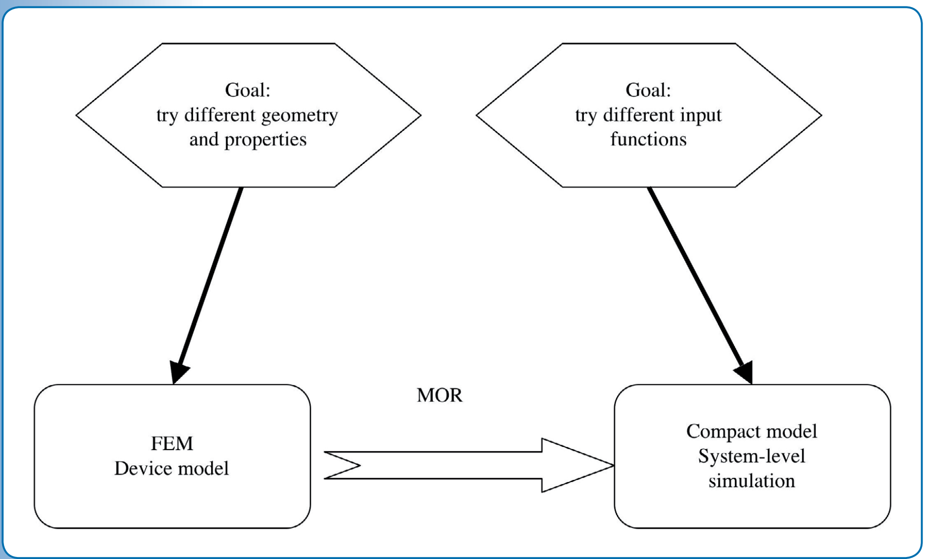

Figure 7:
Use of model reduction during design and system-level simulation.

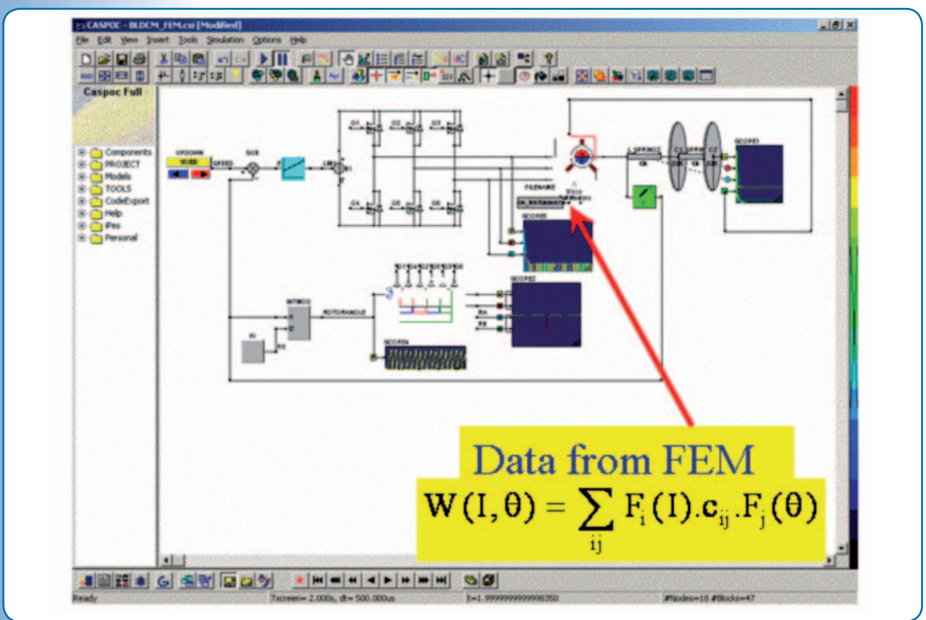

Figure 8:
System simulation of brushless DC motor with parameters from the finite element simulation.

MOR for ANSYS has been used during the optimization process for a vibro-acoustic problem, with stacking sequences of the composite structure as design variables.

As was already mentioned, the method and software presented in this paper are limited to linear dynamic systems only and it seems that there is no way to generalize the approach for an arbitrary nonlinear dynamic system. Fortunately, it is possible to find an effective way to generate macromodels for a particular application by exploring its specific properties. For example, in [12] it was demonstrated how to use co-energy in order to effectively generate a reduced model for a magnetic actuator. This allows us to perform system level simulation with a reduced device model derived directly the finite element simulation (see Fig. 8).

This approach can be used for plungers, of brushless DC motors, permanent magnet synchronous machines, switched reluctance machines as well as relays.

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