

From Finite Elements to System Level Simulation by means of Model Reduction

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Abstract

MATLAB/Simulink, Mathematica, Simplorer and other system level simulation tools can be linked to finite elements models by means of modern model reduction. We give an overview about the method and its value in order to couple finite element and system level simulation with examples made with MOR for ANSYS.

1. Introduction

Many modern manufactured products have complex structure that behavior is controlled by embedded electronics. Hence system level simulation is an important part of the product development. Such simulation includes circuit components combined with device models and the main practical problem here is how to develop an appropriate device model.

Nowadays finite element modeling enjoys widespread use and natural desire is to employ the FE models directly for system level simulation. This is possible in co-simulation, when different tools are coupled together during single dynamic simulation. The difficulty is that the FE models are high dimensional and integration in time in this case is just infeasible.

Common practice for system level simulation is to employ a compact or behavioral device model. Such a model is low-dimensional but it is supposed to approximate the dynamic response with good accuracy. The big problem along this way is evident - how one should actually obtain this model.

Thus there is a gap in simulation practice. On one hand, there is an accurate finite element model that has been already developed; on the other hand, it is still necessary to invest time and efforts to develop a behavioral model for system level simulation. In the other words, one should pay twice: to develop not only a finite element but also a compact model.

The goal of the present paper is to introduce modern model reduction [1] that bridges this gap (see Fig 1). In Section 2 we introduce model order reduction, after that in Section 3 we describe software that applies new algorithms directly to an ANSYS model, and then in Section 4 and 5 we consider several examples to show how modern model reduction is used in practice in different areas of engineering.

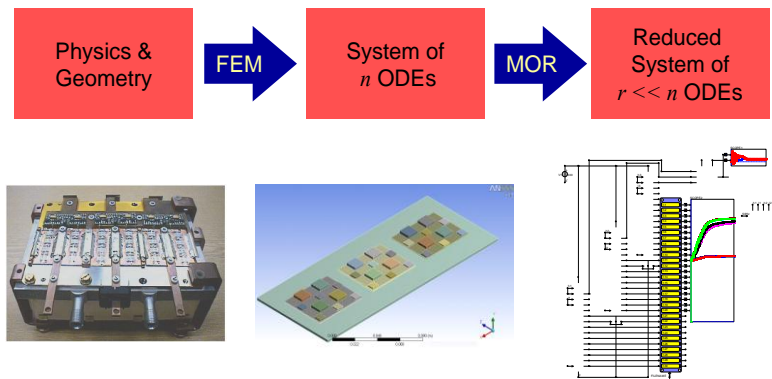


Fig 1: Model order reduction is an efficient means to enable a system-level simulation. Figure shows an example of a compact model for an IGBT module [2].

2. Model Order Reduction

Model reduction is an area of mathematics that in other words can be referred to as approximation of large scale dynamical system [1]. This is a relatively new technology for the finite element community. The concept of model reduction here as such is not new: the mode superposition and the Guyan reduction have been already employed by engineers for long time. However if one compares the book [1] describing the latest achievements from mathematicians with the book [3] that presents a good overview of approaches employed by engineers, he/she will see a big difference. Whereas the methods developed by engineers are mainly based on intuition, the methods developed by mathematicians are based on a strong mathematical background.

Model reduction starts after the discretization of governing partial differential equations when one obtains ordinary differential equations either of the first (Eq 1) or the second (Eq 2) order.

$$\begin{aligned} E\dot{x} + Kx &= Bu \\ y &= Cx \end{aligned} \quad (1)$$

$$\begin{aligned} M\ddot{x} + E\dot{x} + Kx &= Bu \\ y &= Cx \end{aligned} \quad (2)$$

In Eq (1) and (2) M , E , and K are the system matrices and x is the state vector containing degrees of freedom in the finite element model. The main difference from a typical finite element notation is 1) splitting of the load vector to a product of a constant input matrix B and a vector of input functions u and 2) the introduction of the output vector y that contains some linear combinations of the state vector that are of interest in system level simulation.

Inputs and outputs in dynamic systems (1) and (2) affect its dynamic behavior considerably and it is important to take them into account during model reduction. Here is the main

difference between modern model reduction with mode superposition and the Guyan reduction. In order to find the low-dimensional subspace, mode superposition and the Guyan reduction use only the system matrices, while modern model reduction uses all matrices including input and output matrices. Yet, it should be stressed that input functions u do not take part in the model reduction process and they are transferred from the original to the reduced model without any changes. At the same time, the model reduction based on the Arnoldi process that is considered in the present paper allows us to preserve the complete output, that is, the output matrix C can be equal to the unity matrix.

Model reduction is based on an assumption that the movement of a high dimensional state vector can be well approximated by a small dimensional subspace (Fig 2 left). Provided this subspace is known the original system can be projected on it. This is illustrated for the system of the first order in Fig 2 and it can be generalized to the second order systems the same way.

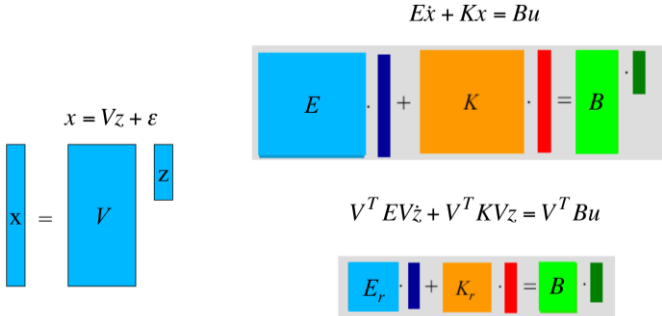


Fig. 2: Model reduction as a projection of the high dimensional system onto the low-dimensional subspace.

The main question is how to find the low dimensional subspace that possesses good approximating properties. In structural mechanics, the subspace formed by the eigenvectors corresponding to the lowest frequencies enjoys widespread use. However, it happens that there are much better choices based on Krylov subspaces [1]. It is actually faster to compute them as compared with the modal analysis and, at the same time, they possess better approximation properties. Another advantage is that the projection subspace remains real valued also for unsymmetric matrices.

The model reduction theory is based on the approximation of the transfer function of the original dynamic system. It has been proved that in the case of Krylov subspaces the reduced system matches moments of the original system for the given expansion point. In

other words, if we expand the transfer function around the expansion point, first coefficients will be exactly the same, as for the original system. Mathematically speaking this approach belongs to the Padé approximation and this also explains good approximating properties of the reduced models obtained through modern model reduction. The detailed description of the algorithm and the theorems proving moment matching properties can be found in the book [1].

The dimension of the reduced model during the model reduction process is controlled by the approximation error specified by the user. Although the model reduction methods based on the Padé approximation do not have global error estimates, in practice it is enough to employ an error indicator [4]. In our experience it is working reasonably well for a variety of finite element models.

Finally it should be mentioned that although the original idea of model reduction was to develop a compact model for system level simulation, the time to run the Arnoldi process is comparable with a couple of static solutions [5]. That is, it is much faster to reduce the original model and perform simulation with the reduced model than to perform dynamic simulation of the original high dimensional model. This implies another use of model reduction as fast solver and this allows us to use model reduction also in the optimization process when the reduced model will be used only once (see section 5).

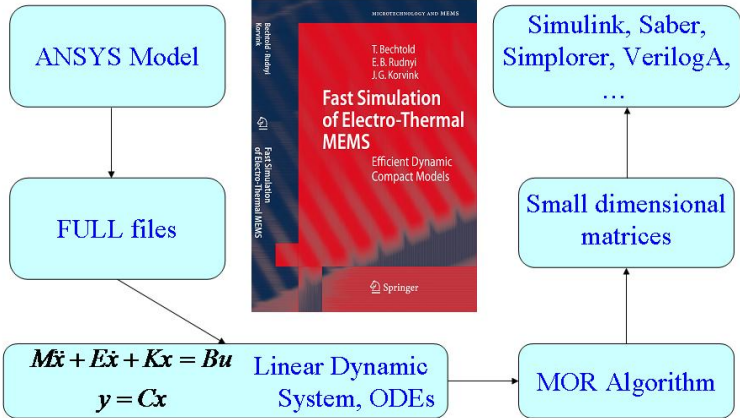


Fig 3: The structure of MOR for ANSYS [5]

3. MOR for ANSYS

The software MOR for ANSYS has been developed at IMTEK, Freiburg University [5][6]. The software reads system matrices from ANSYS FULL files, runs a model reduction algorithm and then writes reduced matrices out (see Fig. 3). The process of generating FULL files in

Workbench is automated through scripting. The reduced matrices can be read directly in MATLAB/Simulink, Mathematica, Python, Simplorer and other system level simulation tools. It is also possible to write them down as templates for the use in Saber MAST, VerilogA and VHDL-AMS.

For the first order systems in the form of Eq (1), the software runs the Arnoldi process directly for the Krylov subspace made from the system matrices E and K and the input matrix B . For the second order systems in the form of Eq (2), there are three options showed in Fig 4.

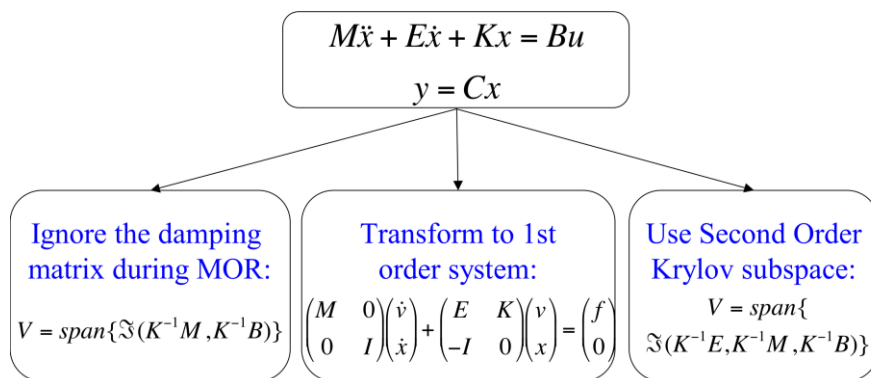


Fig. 4. Model reduction options for a second order system in the form of Eq 2.

First, in the common case of proportional damping

$$E = \alpha M + \beta K \tag{3}$$

the damping matrix can simply be ignored during the process of constructing the projection basis. In this case, only the mass and stiffness matrices together with the input matrix are employed to generate the required Krylov subspace. The damping matrix is projected afterwards and because of Eq (3) it can actually be computed from reduced mass and stiffness matrices. It is worthy to note that in the case of proportional damping, moment matching properties have been proved to hold for any values of α and β [7].

In the general case of nonproportional damping, it is always possible to transform dynamic system (2) to the first order system by increasing the dimension of the state vector twice. The disadvantage here is that a reduced system is obtained in the form of the first order system and that computational requirements increase because of the increase in the dimension of the state vector. The use of the second order Krylov subspaces [8][9] removes both disadvantages mentioned before.

MOR for ANSYS uses well-known solvers MUMPS [10] und TAUCS [11] together with the METIS ordering [12] and the optimized BLAS, implements the error indicator to choose the dimension of the reduced model [4], and in addition to the conventional block Arnoldi algorithm [1] employs the superposition Arnoldi [13]. The latter is superior over the block Arnoldi in the case of many inputs [13].

MOR for ANSYS has been used for a variety of finite element models: electro-thermal MEMS, structural mechanics, piezoelectric actuators for control, pre-stressed small-signal analysis for RF-MEMS, thermomechanical models, and acoustics including fluid-structure interactions. There are many MOR for ANSYS related publications: 1 book, 3 book chapter, 4 theses, 16 journal papers and over 60 conference papers. The full list of publications is available at <http://modelreduction.com/publicationsByYears.html> and in the next sections a few selected applications will be presented.

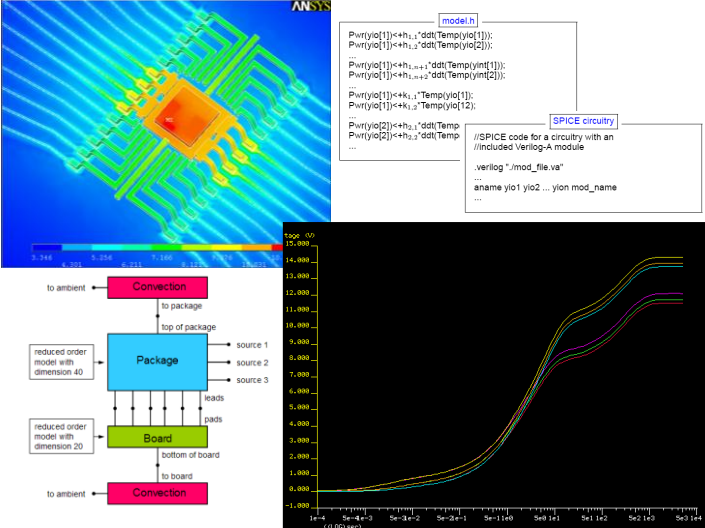


Fig 5: Dynamic compact thermal model of a package [4]. Figure shows a stationary solution, a block scheme for system level simulation, fragments of the implementation in VerilogA and results at the system level.

4. Employing Model Reduction to Generate Dynamic Compact Models

We start with electrothermal simulation of IGBT in a hybrid vehicle (see Fig 1) [2]. An electrical model of IGBT depends on temperature and the latter should be available during system level simulation. An IGBT module shown in the middle of Fig 1 contains three DCPs with 12 IGBTs and 18 diodes, which define 30 heat sources. With the finite element model in ANSYS one obtains accurate temperature distribution that also takes into account thermal cross talk. MOR for ANSYS generates small matrices and one uses them at the system level

for electrothermal simulation [2]. Examples with electrothermal MEMS devices are available in the book [14] and in Fig 5 there is an example of a package from Freescale [15] where system level simulation has been performed in VerilogA.

Although model reduction was developed for linear dynamical models [1], the nonlinearity in the input function can be treated without any changes. As was already mentioned, the input function does not take part in the model reduction process and one employs it without changes. What is necessary is only to estimate the state required to evaluate the input function in the reduced model. Fig 6 presents an example when the heat generation depends on temperature as the resistivity of the heater changes with the temperature [16].

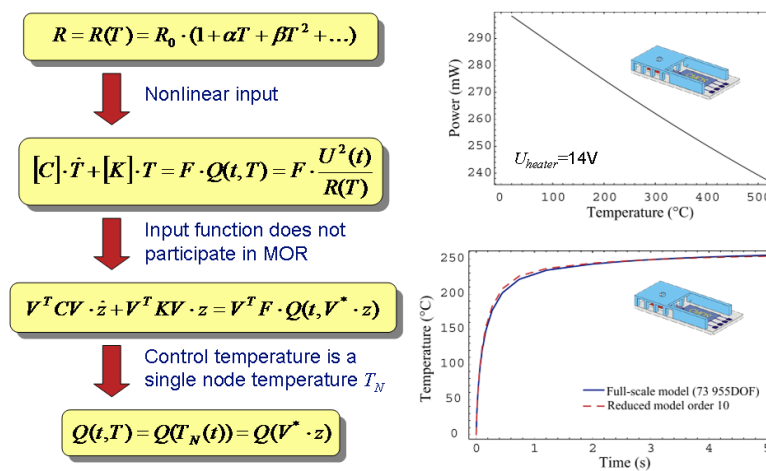


Fig. 6. Model reduction for an electrothermal model of a microhotplate with temperature dependent resistivity of the heater [16].

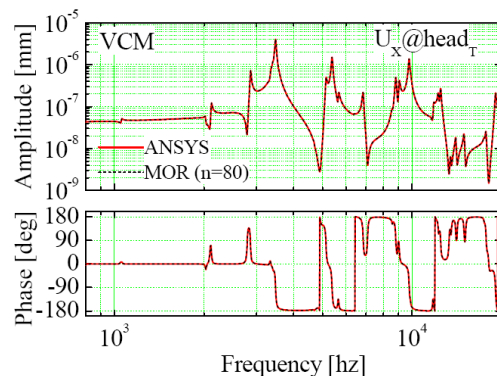
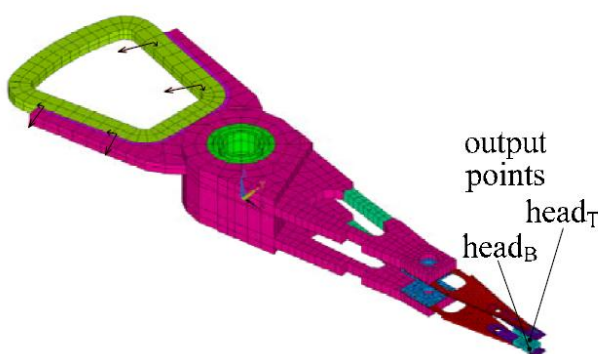


Fig. 7. HDD actuator and suspension system [17]. The reduced model of dimension 80 approximates the harmonic response of the ANSYS model in the range from 800 to 20000 Hz with an error within the line thickness.

MOR for ANSYS is also applicable for structural models. In Fig 7 there is an example of a HDD actuator and suspension system [17]. The reduced model of dimension 80 approximates the harmonic response of the ANSYS model in the range 0 to 20000 Hz with an error within the line thickness. The dimension 80 is well-suited for system level simulation and here is huge difference with the case of co-simulation where the original ANSYS model with more than 70000 degrees of freedom would be employed. Another example from the area of machining tools is given in Fig 8 where there is also a vision from IWF/inspire at ETH for the use of model reduction in the development of a machining tool.

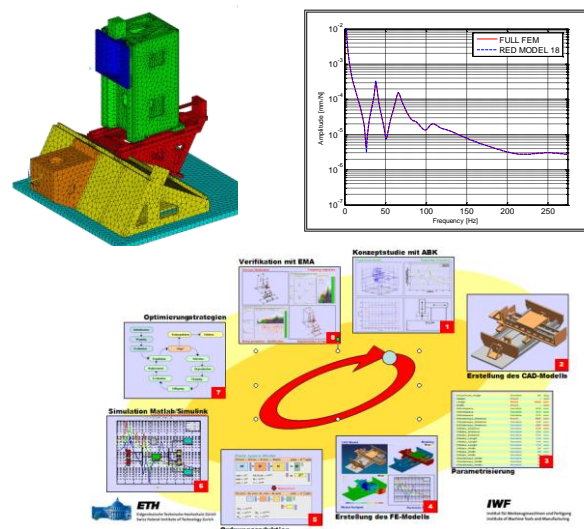


Fig 8: Harmonic response of the Tool Center Pointer for a machining center and a vision of IWF/inspire at ETH for the use of model reduction to develop a machining tool.

5. Employing Model Reduction as Fast Solver

It was already mentioned that the model reduction process with MOR for ANSYS is much faster than a dynamic simulation in ANSYS with the original model. This way it is possible to use MOR for ANSYS as a fast solver for transient or harmonic simulation. Let us consider a model developed at Voith Siemens Hydro Power Generation (see Fig 9). The goal of the simulation is to study the dynamic excitation of turbine rotors by rotating pressure field caused by rotor-stator interaction. A reduce model of dimension 100 very accurately approximates the original ANSYS model. However, the time to generate the reduced model and make harmonic simulation with it is orders of magnitude faster than to perform harmonic response simulation in ANSYS.

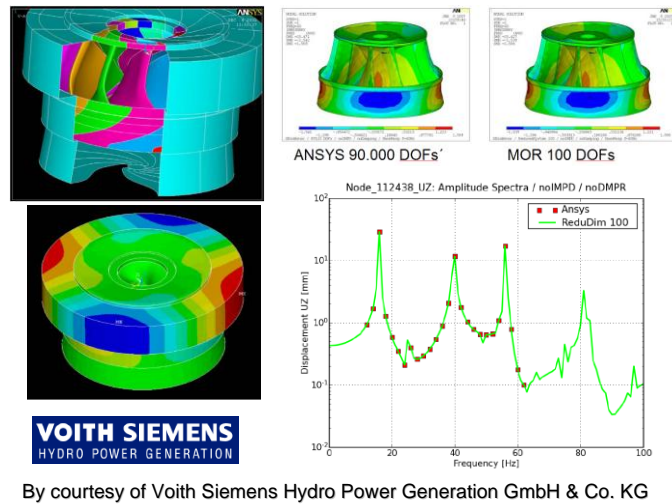


Fig 9: Model reduction for a FSI problem. Figure shows dynamic excitation of turbine rotors by rotating pressure field caused by rotor-stator interaction.

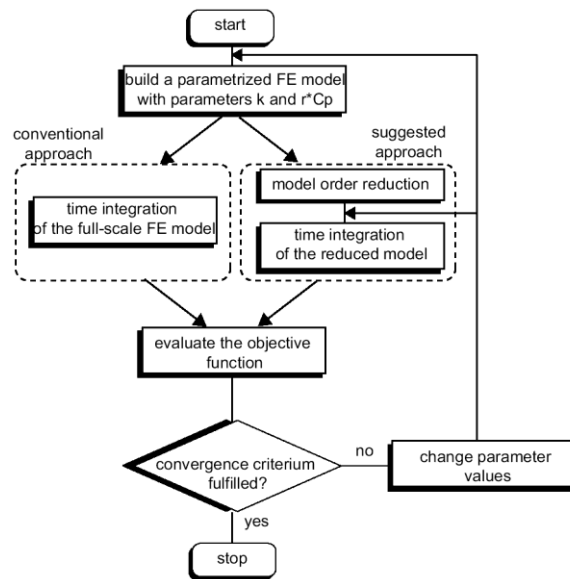


Fig. 10. The use of model reduction in the optimization loop as fast solver.

This allows us to employ model reduction as a fast solver in the optimization process (see Fig. 10). The use of MOR for ANSYS for the optimization of a microaccelerometer is documented in [18] and for structural acoustic optimization to improve acoustic characteristics of a vehicle (NVH – Noise, Vibration, Harshness) in [19]. Fig 11 shows the results of the optimization of the composite stacking sequence in order to reduce the noise pressure level. Here model reduction has allowed researches to speed up harmonic simulation for each iteration and thus for the whole optimization by a factor 50.

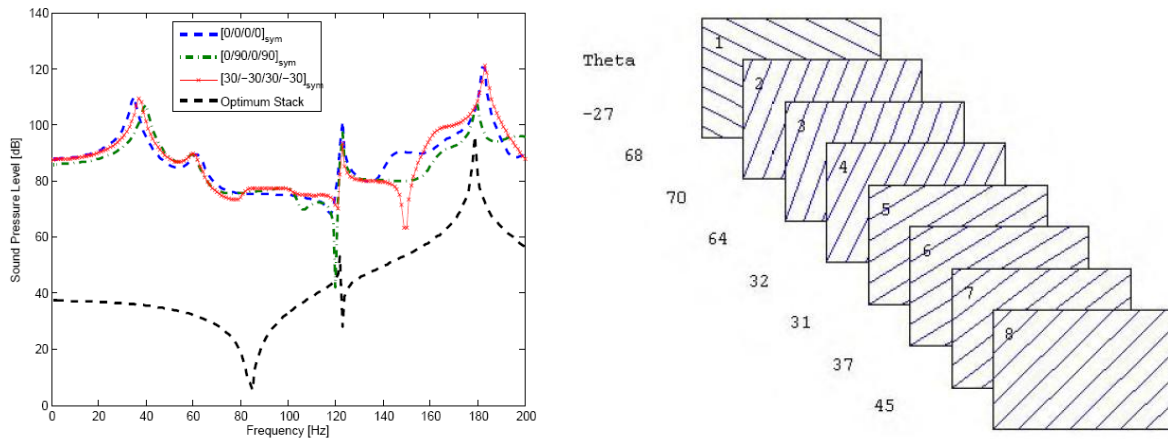


Fig 11: A comparison of Arnoldi predicted fluid pressure for composite stacking sequences: $[0/0/0/0]_{sym}$, $[0/90/0/90]_{sym}$, $[30/-30/30/-30]_{sym}$ and optimum stacking sequence $[153/68/70/64/32/31/37/45]$ obtained by LHS/MADS optimization. On the right is the optimal stacking sequence [19].

6. Conclusion

We have shown that model reduction is a perfect tool to generate accurate reduced models directly from the finite element models. It should be mentioned that MOR for ANSYS is applicable for any linear model developed in ANSYS either as a tool to automatically generate a compact dynamic model for system level simulation or a fast solver for dynamic simulation. It has been also shown that nonlinearity in the input function can be treated in the present framework without any changes. When the system matrices are nonlinear it is possible to linearize the model around the operation point. Such an approach for electromechanical models of RF resonators is presented in [20]. Alternatively when the nonlinearity is weak the Krylov subspace method can be generalized to include quadratic and cubic effects [21]. Finally moment matching can be also generalized to the case when it is necessary to preserve some parameters from the system matrices in the reduced model as symbols [22][23].

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