Order Reduction of Second Order Systems with Proportional Damping
ALA 2006

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A common result of modeling (for instance by finite element methods) in some fields like Electrical Circuits and Micro-Electro-Mechanical Systems (MEMS) is a large number of *second* order differential equations. The most practical solution to handle such large scale models is offered by order reduction. In reduced order modeling of such systems, not only the behavior of the original system should be approximated in the frequency range of interest, but also it is advisable to preserve the structure of the original model. We consider the second order system of the form,

$$\left\{ \begin{array}{l} \mathbf{M}\ddot{\mathbf{z}}(t) + \mathbf{D}\dot{\mathbf{z}}(t) + \mathbf{K}\mathbf{z}(t) = \mathbf{G}\mathbf{u}(t), \\ \mathbf{y}(t) = \mathbf{L}\mathbf{z}(t). \end{array} \right. \tag{1}$$

The matrices $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are called mass, damping and stiffness matrices, respectively.
In [3] and others, the Krylov subspaces were extended to the so called *Second Order Krylov Subspaces* to reduce second order systems by applying a projection directly to the original model and matching the moments.
In many second order systems, the damping matrix is modeled as a linear combination of the mass and stiffness matrices ($\mathbf{D} = \alpha\mathbf{M} + \beta\mathbf{K}$) which is known as proportional or Rayleigh damping. For this special set of second order systems, the reduced order modeling using a second order Krylov method may be simplified. In [2, 1], it is proposed to calculate the projection matrices by neglecting the damping matrix, and then apply the projection to the system with damping. The main problem in these references is that the moment matching property is only proved for the case that $\mathbf{D} = \mathbf{0}$ and the quality of the damped reduced systems is only demonstrated empirically through examples and not based on moment matching.
In this talk, we first show that for systems with a proportional damping matrix, the damping matrix does not contribute to the projection matrices. It is proved that, the corresponding subspaces are equal to some standard Krylov subspaces, independent of the values of α and β and consequently independent of $\mathbf{D}$.
This fact simplifies the reduction procedure and allow us to use the standard numerical algorithms implemented for moment matching in state space that are generally more robust than the second order Krylov algorithms.
It should be noted in order reduction of proportionally damped systems by applying a direct projection, the property of proportional damping is preserved in the reduced system. In other words, if $\mathbf{D} = \alpha\mathbf{M} + \beta\mathbf{K}$ then $\mathbf{D}_r = \alpha\mathbf{M}_r + \beta\mathbf{K}_r$ in the reduced model.

References

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